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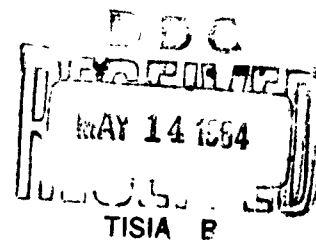
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A Redundancy Technique for Improving the Reliability of Digital Systems

by
J. K. Knox-Seith

December 1963



Technical Report No. 4816-1

Prepared under
Office of Naval Research Contract
Nonr-225(44), NR 375 865

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A REDUNDANCY TECHNIQUE FOR IMPROVING THE
RELIABILITY OF DIGITAL SYSTEMS

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J. K. Knox-Seith

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Solid-State Electronics Laboratory
Stanford Electronics Laboratories
Stanford University Stanford, California

ABSTRACT

The purpose of this report is to give a quantitative evaluation of the improvement in reliability which can be achieved in a digital system by the use of redundancy and restoring organs.

Three measures of reliability are considered:

1. The probability of system survival $P(T)$ for a given mission time T .
2. The mean time to failure for the system; $MTF = \int_0^{\infty} P(t) dt$.
3. The useful life T_{Δ} of the system. T_{Δ} is defined as the maximum mission time for which $P(T) \geq 1-\Delta$.

Two types of restoring organs, majority vote takers and adaptive vote takers are considered.

For the case of majority vote takers simple expressions have been developed for the approximate relationships between the amount of redundancy, the number of vote takers, and the corresponding improvement in system reliability. The analysis includes the case of redundant nonperfect vote takers, and the optimum number of imperfect vote takers (of known reliability) to be used in any system has been established.

For the case of perfect vote takers in a highly redundant system, we find that the system MTF and T_{Δ} increase almost proportionally with the number of vote takers employed. The expressions developed in the text can readily be used to evaluate the trade-off between the amount of redundancy and the number of vote takers required to achieve a desired improvement in reliability.

Furthermore, we have investigated the improvement in reliability which can be achieved by using adaptive vote takers as the restoring organ.

For systems of redundancy higher than three the adaptive vote taker is a more efficient restoring organ than the majority vote taker. However, we find that the reliability which can be achieved in a redundant system by using a given number of adaptive vote takers can often be equalled or exceeded by using about 10 times as many majority vote takers. At the present time, there is no simple technique for realizing adaptive

vote takers, whereas majority vote takers with up to about 9 inputs are relatively easy to implement. Thus, for the time being the use of majority vote takers appears more practical than the use of adaptive vote takers.

It is concluded that while the use of redundancy and restoring organs can substantially increase the MTF of a digital system, the technique is much more effective in increasing the useful life T_{Δ} (for $\Delta \ll 1$) of a system. Thus, this technique will be most useful in the case of a system that must operate with an exceedingly small probability of failure for a relatively short period of time.

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EXPLANATION OF SYMBOLS

P, p	the probability that a system or a circuit is operating properly. p is used for individual circuits and for single-stage systems. P is used for systems containing m stages, where m usually is greater than one. Whenever we want to emphasize that P depends on time, we will write $P(t)$. A set of subscripts on P or p is used to distinguish between various cases as is indicated in detail below.
Q, q	the probability that a circuit or a system has failed. (Once a circuit has failed, it is assumed to remain inoperative.) The same comments as stated above for P apply to Q .
n	each majority group contains $(2n+1)$ identical circuits.
m	the number of stages contained in a system using majority vote takers (also the number of nonredundant majority vote takers)
m_a	the number of stages in a system using adaptive vote takers
M	the optimum number of stages for a system using redundant unreliable majority vote takers
$\lambda = \lambda_o$	failure rate for a nonredundant circuit (or subsystem)
$L = m\lambda$	failure rate for a nonredundant system containing m circuits (or m subsystems)
λ_v	failure rate for a majority vote taker
$p_o(t)$	probability that a circuit will operate successfully from time 0 to t assuming that the circuit operated properly at $t = 0$
$q_o(t) = 1 - p_o(t)$	probability that a circuit failed in the period 0 to t
$q_n(t)$	probability that a majority group containing $(2n+1)$ circuits failed in the period 0 to t
$p_n(t) = 1 - q_n(t)$	

EXPLANATION OF SYMBOLS (Continued)

$Q_o(t)$ } probability that a nonredundant system containing
 $P_o(t) = 1 - Q_o(t)$ } m circuits failed during the period 0 to t

$Q_n(t)$ } probability that a redundant system containing m
 $P_n(t) = 1 - Q_n(t)$ } majority groups each with $(2n + 1)$ identical
circuits failed in the period 0 to t

$q_{na}(t)$ } probability that an adaptive majority group
 $p_{na}(t) = 1 - q_{na}(t)$ } containing $(2n + 1)$ circuits failed in the
period 0 to t

$Q_{na}(t)$ } probability that a system containing m_a adaptive
 $P_{na}(t) = 1 - Q_{na}(t)$ } majority groups each with $(2n + 1)$ circuits will
fail in the period 0 to t

$q_v(t)$ } probability that a majority vote taker will fail in
 $p_v(t) = 1 - q_v(t)$ } the period 0 to t

$T_{\Delta o}$ that period of time for which a nonredundant system
can operate with the probability of system failure
being less than or equal to Δ ($\Delta \ll 1$)

$T_{\Delta n}$ that period of time for which a system containing m
majority groups each with $(2n + 1)$ circuits can
operate with the probability of system failure being
less than or equal to Δ

T_1 lower bound on the mean time to failure (MTF) for a
redundant system

I. INTRODUCTION

The purpose of this report is to give a quantitative evaluation of the improvement in reliability which can be achieved in a digital system by the use of redundancy and restoring organs.

Two types of restoring organs are considered, and the relative merits of these organs are discussed. The types of restoring organs being considered are majority vote takers (MVT) and adaptive vote takers (AVT).

A. THE FAILURE MODEL

In this report we are primarily concerned with the effect of circuit failures on system performance. Ideally, the output from a properly operating digital circuit is completely determined by the preceding sequence of input digits, whereas the output from a circuit that has failed is independent of the input to the circuit. In a practical situation a circuit will usually be close to one or the other of these two conditions; that is, a circuit will generally either have an error rate which is many orders of magnitude less than 1 or an error rate in the order of $\frac{1}{2}$.

It is assumed that a failed circuit can only be restored to proper operation by being repaired. We shall find it convenient for part of the analysis to assume that a failed circuit always gives the complement of the desired output. The implications of this assumption are discussed in Chapter II where it is pointed out that this assumption will lead to a pessimistic estimate of the improvement in reliability to be achieved by the use of redundancy and restoring organs.

It is furthermore assumed that the circuit failures are independent, and that the number of circuit failures in a given length of time is Poisson distributed. It can be shown [Ref. 1] that this in general is the failure distribution to be expected in large electronic systems.

B. REDUNDANCY AND MAJORITY VOTE TAKERS

One technique for improving the reliability of a digital system

containing m digital binary subsystems (or circuits) is to replace each subsystem by a group of $2n + 1$ identical subsystems which have the inputs connected in parallel. The binary outputs from such $2n + 1$ circuits are fed to a "majority vote taker," so that the overall output from the group of $2n + 1$ circuits will be that output shown by the majority of the circuits (Fig. 2.6). Such a group of identical binary circuits plus associated "vote taker" is referred to as a "majority group." It is seen that a majority group will give the desired output if more than half of the $(2n + 1)$ circuits in the group show the correct output.

A number of recent papers have investigated various aspects of the use of redundancy and majority logic for improving the reliability of digital systems. [See for example Refs. 2-6.]

In Chapters II and III of this report we investigate the improvement in reliability which can be achieved by dividing a digital system into m circuits (or subsystems) and replacing each circuit by $2n + 1$ identical circuits followed by a majority vote taker. Simple expressions are developed for the approximate relationships between added system complexity (i.e., the amount of redundancy and the number of vote takers) and the corresponding improvement in system reliability. These simple relationships expressing quantitatively the trade-off between system complexity and system reliability are believed to be new.

C. REDUNDANCY AND ADAPTIVE VOTE TAKERS

For a fixed amount of redundancy (greater than 3) and a fixed number of vote takers, the reliability of a system can be further improved if the majority vote takers are replaced by adaptive vote takers. By comparing the output from each individual circuit with the output from the vote taker, it is possible to estimate the error rate for the individual circuits. An optimum voting procedure can then be established in which the most reliable circuits carry more weight in the voting than the less reliable circuits. In its simplest form the adaptive vote taker gives either weight one or weight zero to a circuit; that is, initially all circuits carry the same weight, until the error rate of one circuit increases beyond some threshold level, in which case that circuit is

eliminated from the vote taker. The adaptive vote taker will thus eliminate the circuits as they fail, and the "adaptive majority group" may then operate as long as at least two circuits in the group are operating properly. The reliability which can be achieved by this technique is investigated in Chapter IV.

D. MEASURES OF RELIABILITY

Three measures of reliability for a digital system are considered:

- 1) Reliability of a system is frequently defined as the probability $P(t)$ that the system will work successfully (i.e., without failure) for a given mission time t , assuming that the system was operating at the start of the mission. $P(t)$ is referred to as the probability of survival. $Q(t) = 1 - P(t)$ is then the probability that the system will fail in the given period of time t . The improvement in reliability of a system employing redundancy relative to the reliability of the nonredundant system can then be defined as

$$I_1(t) = \frac{\text{probability of survival for redundant system}}{\text{probability of survival for nonredundant system}} = \frac{P_n(t)}{P_o(t)} \quad (1.1)$$

Alternatively, the improvement could be expressed in terms of the probability of failure for the two systems:

$$I_2(t) = \frac{\text{probability of failure for nonredundant system}}{\text{probability of failure for redundant system}} \\ = \frac{1 - P_o(t)}{1 - P_n(t)} = \frac{Q_o(t)}{Q_n(t)} \quad (1.2)$$

- 2) The mean time to failure (MTF) of a system may be taken as the

measure of system reliability. By definition $MTF = \int_0^{\infty} P(t) dt$;

the relative improvement in reliability obtained by the use of redundancy can then be expressed as

$$I_3 = \frac{\text{MTF for redundant system}}{\text{MTF for nonredundant system}} \quad (1.3)$$

- 3) As a third measure of system reliability we define "the useful life" T_Δ of a system to be the longest mission time for which the probability of survival is greater than or equal to $1 - \Delta$. Alternatively, we may say that T_Δ is the longest mission time for which the probability of failure is no greater than Δ . It follows that

$$Q(T_\Delta) = \Delta \quad \text{and} \quad Q(t) = 1 - P(t) \leq \Delta \quad \text{for all} \quad t < T_\Delta.$$

The corresponding improvement factor is defined as

$$I_4(\Delta) = \frac{T_\Delta \text{ for the redundant system}}{T_\Delta \text{ for the nonredundant system}} = \frac{T_{\Delta n}}{T_{\Delta o}} \quad (1.4)$$

E. OUTLINE OF REPORT - RESULTS

In Chapter II we consider a redundant system using m perfect majority vote takers as the restoring organs.

We first derive expressions for the probability of survival $P(t)$ and the probability of failure $Q(t)$ as a function of time. Curves of $P(t)$ and $Q(t)$ are given for various values of m and n (m vote takers, $2n + 1$ redundant circuits).

Next we derive approximations for T_Δ when $\Delta \ll 1$. T_Δ is given in terms of the MTF for the nonredundant system and as a function of the number of vote takers and the amount of redundancy.

Finally, we establish lower bounds on the MTF for the redundant system. These bounds are given in terms of the MTF for the nonredundant system and are functions of the amount of redundancy and the number of vote takers.

We find that the introduction of redundancy changes the shape of the function $P(t)$ significantly. For the nonredundant system, $P(t)$ changes gradually from one to zero, as t increases. For a redundant system, $P(t)$ tends to be either close to one (for small t) or close

to zero (for large t), with a relatively steep transition between these two regions (see Figs. 2.3, 2.4, 2.7 and 2.8). As the amount of redundancy is increased, the separation of $P(t)$ into two regions, close to one and zero respectively, becomes increasingly more pronounced. For systems with large redundancy, the time T_{Δ} , for which the probability of survival is close to one, will therefore be close to the MTF.

In the case of perfect vote takers we find that T_{Δ} and the MTF increase with the number of vote takers as $m^{n/n+1}$ (m being the number of vote takers and $2n + 1$ being the number of redundant circuits.) Thus, T_{Δ} and the MTF will, in a system with large redundancy, increase almost proportionally with the number of vote takers.

Tables 1 and 2 give examples of the improvement in reliability to be achieved by the use of redundancy and majority logic. For example, the relative increase in MTF obtained by using a redundancy of 5 and $m = 100$ vote takers is found to be 8. For $m = 1000$ the relative increase in MTF would be 39 (with a redundancy of 5).

The relative increase in useful life T_{Δ} which can be achieved by this technique is much more impressive. For example, if the permissible probability of failure is $\Delta = 10^{-2}$, then with a redundancy of 5 and with $m = 100$ vote takers, a relative increase in T_{Δ} of 215 times will be achieved. Under the same condition, if $m = 1000$ the relative improvement in T_{Δ} would be 1000 times. For a smaller value of Δ the relative increase in T_{Δ} would be still greater, since the relative increase in T_{Δ} is proportional with $(1/\Delta)^{n/n+1}$.

In Chapter III we consider the case of non-perfect majority vote takers. We first establish a condition on the failure rate of the majority vote taker relative to the failure rate of the system in order that the vote taker can be considered ideal.

In the case when the vote takers cannot be considered ideal the use of redundant vote takers is suggested. The expressions derived in Chapter II for T_{Δ} and for the lower bound on the MTF are modified in Chapter III to include the effect of unreliable redundant vote takers.

As one would expect it is found that the system reliability decreases if too many unreliable vote takers are inserted in the system. An

expression for the optimum number M of unreliable vote takers to be used in a given system is established. It is concluded that in most situations we will be limited by practical considerations to use far fewer than M vote takers. It should be pointed out that the optimum number of redundant vote takers to be used does not depend on which of the 3 measures of reliability is used. On the other hand, the condition for a vote taker to be considered ideal depends strongly on the measure of reliability that we use.

For a given system the expressions developed in Chapters II and III permit a simple quantitative evaluation of the trade-off between system complexity (i.e., equipment redundancy and the number of majority vote takers) and system reliability.

It is found that redundancy and majority vote takers can be used to substantially increase the MTF of a large digital system; that is a system which can conveniently be divided into a large number of binary subsystems. Furthermore, it is found that this technique is much more effective in increasing that period of time for which the probability of system failure is close to zero. Thus, this technique will be most effective in the case of a system which must operate with a very small probability of failure during a relatively short mission time.

In Chapter IV we consider the use of adaptive vote takers as the restoring organ in a redundant system. We establish expressions for the probability of failure $Q_{na}(t)$ for a redundant system using adaptive vote takers. Curves showing $Q_{na}(t)$ as a function of t are given for redundancy in the range 5 to 65.

Finally, the reliability achieved in a redundant system by using adaptive vote takers is compared with the reliability achieved by using majority vote takers. We find that under a wide range of conditions the use of approximately 10 MVT's instead of each AVT will result in a system of superior reliability.

II. MAJORITY LOGIC WITH PERFECT VOTE TAKERS

In this chapter we consider a redundant digital system containing m restoring organs; each restoring organ being a perfect infallible majority vote taker.

We derive expressions for the probability of survival $P(t)$ and the probability of failure $Q(t)$ for a redundant system containing m majority vote takers. Plots of $P(t)$ and $Q(t)$ are given for a wide range of m and n .

Next we derive an approximation for "the useful life" T_{Δ} ($\Delta \ll 1$) as a function of a) the failure rate of the corresponding nonredundant system, b) the redundancy of the system, c) the number of vote takers used, and d) Δ . This approximation (2.22) is useful for evaluating the trade-off between the amount of redundancy and the number of vote takers required to achieve a desired reliability.

Furthermore, we establish lower bounds on the MTF for the redundant system, and finally, we compare the reliability for the nonredundant system with that of the redundant system in order to determine the improvement achieved by the use of redundancy and majority logic.

Expressions for the improvement in reliability are given in (2.24) and in (2.32). Numerical values of the reliability improvement for a wide range of n and m are given in Tables 1 and 2.

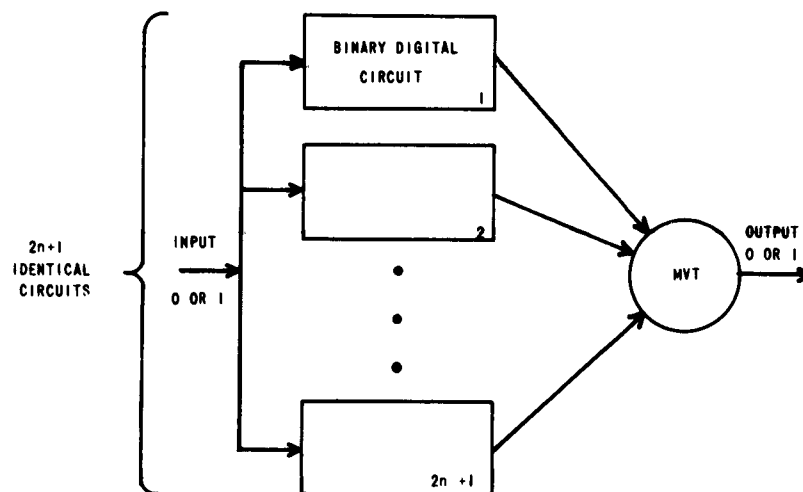
A. MAJORITY LOGIC WITH ONE PERFECT VOTE TAKER - THE MAJORITY GROUP

First, consider a nonredundant binary system as shown in Fig. 2.1A. This system has a binary input and a binary output. If the system is working properly, the output at any time will be 1 or 0 depending in some specified way on the sequence of inputs up to that time. Let p_o denote the probability that the system is operating properly, and let $q_o = 1 - p_o$ be the probability that the system is not operating properly.

In the following we are going to assume that when the system is not operating properly it has as an output the complement of the correct output. (The implication of this assumption is discussed in connection with formula 2.3 below.)



A. Nonredundant digital circuit



B. Majority group

FIG. 2.1. NONREDUNDANT DIGITAL CIRCUIT AND CORRESPONDING MAJORITY GROUP.

Next consider the system of Fig. 2.1B. Here a number of systems identical to the one shown in Fig. 2.1A are driven in parallel; i.e., they all receive the same input. If they were all operating properly, they would all produce the same binary output. The output from the overall system will be determined by a "majority vote taker"; that is, the overall system will give the output which is given by the majority of the individual circuits. This means that the overall system will give the correct output when less than half of the individual circuits have failed.

If the probability of failure for the individual circuits is q_o , then the probability that exactly i circuits out of $2n + 1$ circuits will have failed is

$$\text{Probability (exact } i \text{ failures)} = \binom{2n+1}{i} q_o^i p_o^{2n+1-i} \quad (2.1)$$

where

$$\binom{2n+1}{i} = \frac{(2n+1)!}{i!(2n+1-i)!} = \binom{2n+1}{2n+1-i} \quad (2.2)$$

It is then seen that the probability of failure q_n for the overall "majority group" shown in Fig. 2.1B will be

$$q_n = \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} p_o^{2n+1-i} q_o^i \quad (2.3)$$

assuming an ideal vote taker.

Equation (2.3) is based on the assumption that all the circuits which have failed give the complement of the desired output. If the digital output is represented, for example, by an analog voltage (e.g., one corresponds to +10 V and zero corresponds to -10 V), then it would actually be more realistic to assume that when the digital system has failed then the output may be anywhere between "zero" and "one" (e.g., between -10 V and +10 V). Under that assumption it may be possible to have more than n circuit failures and still achieve the

correct output. It is, however, unlikely that the majority group will be consistently correct when more than n circuits have failed (if n is small). The assumption that a circuit which has failed shows the complement of the desired output is thus a "worst case" assumption, and the probability of failure which we find for the redundant system based on this assumption will, if anything, be too large.

The probability of success (no failure) for the majority group is

$$p_n = \sum_{i=0}^n \binom{2n+1}{i} q_o^i p_o^{2n+1-i} = 1 - q_n \quad (2.4)$$

Figure 2.2 shows p_n as a function of q_o for various values of n . It is seen from Fig. 2.2 that when $q_o < \frac{1}{2}$ the majority group has a probability of success p_n which is greater than that of the individual circuit. For values of $q_o > \frac{1}{2}$ the probability of success for the majority group is less than that of the individual circuit.

Next we will investigate the probability of failure as a function of time. We will assume that the circuit failures are independent and that the number of circuit failures in a given length of time is Poisson distributed. It can be shown [Ref. 1] that this failure distribution in general is to be expected for a large system, i.e., a system containing many components.

It then follows that the time between failures will be exponentially distributed, and we can write the following expression for the probability p_o that a specific circuit has not failed in the time period 0 to t :

$$p_o = p_o(t) = e^{-\lambda t} = 1 - q_o(t) \quad (2.5)$$

where λ is the failure rate and $1/\lambda$ is the MTF for the type of circuit in question. We will also refer to $p_o(t)$ as the probability of survival.

Inserting this expression for $p_o(t)$ into the expression (2.4) we find $p_n(t)$ the probability of survival for a majority group as a function of time:

$$p_n(t) = \sum_{i=0}^n \binom{2n+1}{i} (1 - e^{-\lambda t})^i (e^{-\lambda t})^{2n+1-i} \quad (2.6)$$

Figure 2.3 shows p_n versus time for various values of n .

Recall that the mean time to failure for the system is given by

$$MTF = \int_0^{\infty} p(t) dt$$

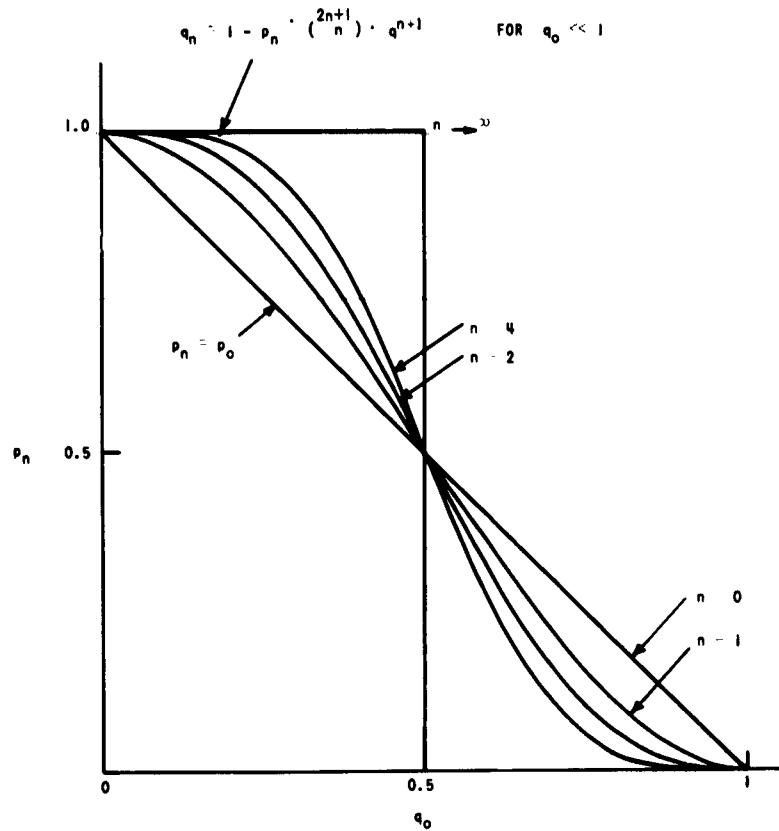


FIG. 2.2. PROBABILITY OF SUCCESS p_n FOR A MAJORITY GROUP CONTAINING $2n + 1$ CIRCUITS, SHOWN AS A FUNCTION OF THE PROBABILITY OF FAILURE q_0 FOR THE INDIVIDUAL CIRCUIT.

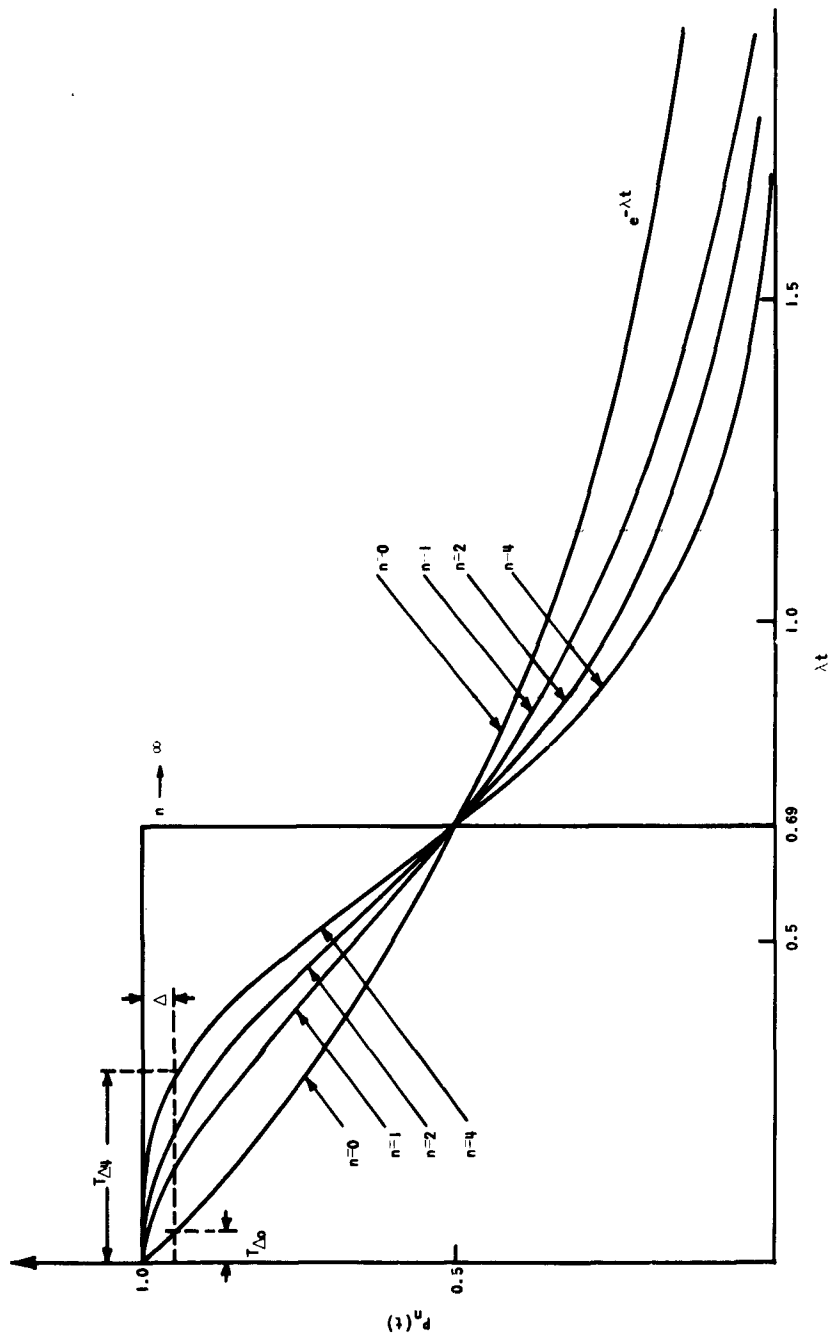


FIG. 2.3. PROBABILITY OF SURVIVAL $p_n(t)$ FOR A MAJORITY GROUP CONTAINING $2n + 1$ CIRCUITS, SHOWN AS A FUNCTION OF λt .

It is then seen from Fig. 2.3 that the MTF of the system actually decreases as n increases, and in the limit as $n \rightarrow \infty$ the MTF of the majority group is 0.69 times the MTF of the nonredundant circuit. However, it is also seen that for large n the probability of failure is either very small (namely, when $t < 1/\lambda$ 0.69) or close to one (when $t > 1/\lambda$ 0.69).

Introducing (2.5) into (2.3) we get

$$q_n(t) = \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} (1 - e^{-\lambda t})^i (e^{-\lambda t})^{2n+1-i} \quad (2.7)$$

Figure 2.4 shows $q_n(t)$ as a function of λt for n in the range 0 to 32. Note how the steepness of the curves increases with increasing redundancy.

To explore the behavior of $q_n(t)$ in the range where the probability of failure is much smaller than one we expand the expression for $q_n(t)$ around $t = 0$ and drop all higher order terms. We find

$$q_n \doteq \binom{2n+1}{n} q_0^{n+1} \quad \text{when} \quad q_0 \ll 1 \quad (2.8)$$

and

$$q_0 = (1 - p_0) = 1 - e^{-\lambda t} \doteq \lambda t \quad \text{when} \quad \lambda t \ll 1 \quad (2.9)$$

so that

$$q_n(t) \doteq \binom{2n+1}{n} (\lambda t)^{n+1} \quad \text{for} \quad \lambda t \ll 1 \quad (2.10)$$

Figure 2.5 shows $q_n(t)$ vs λt for $\lambda t \ll 1$ and for $0 \leq n \leq 4$.

B. MAJORITY LOGIC WITH m PERFECT VOTE TAKERS

In the following we shall investigate the improvement in reliability which can be achieved if several restoring organs (majority vote takers) are used within the redundant system. Consider the nonredundant system

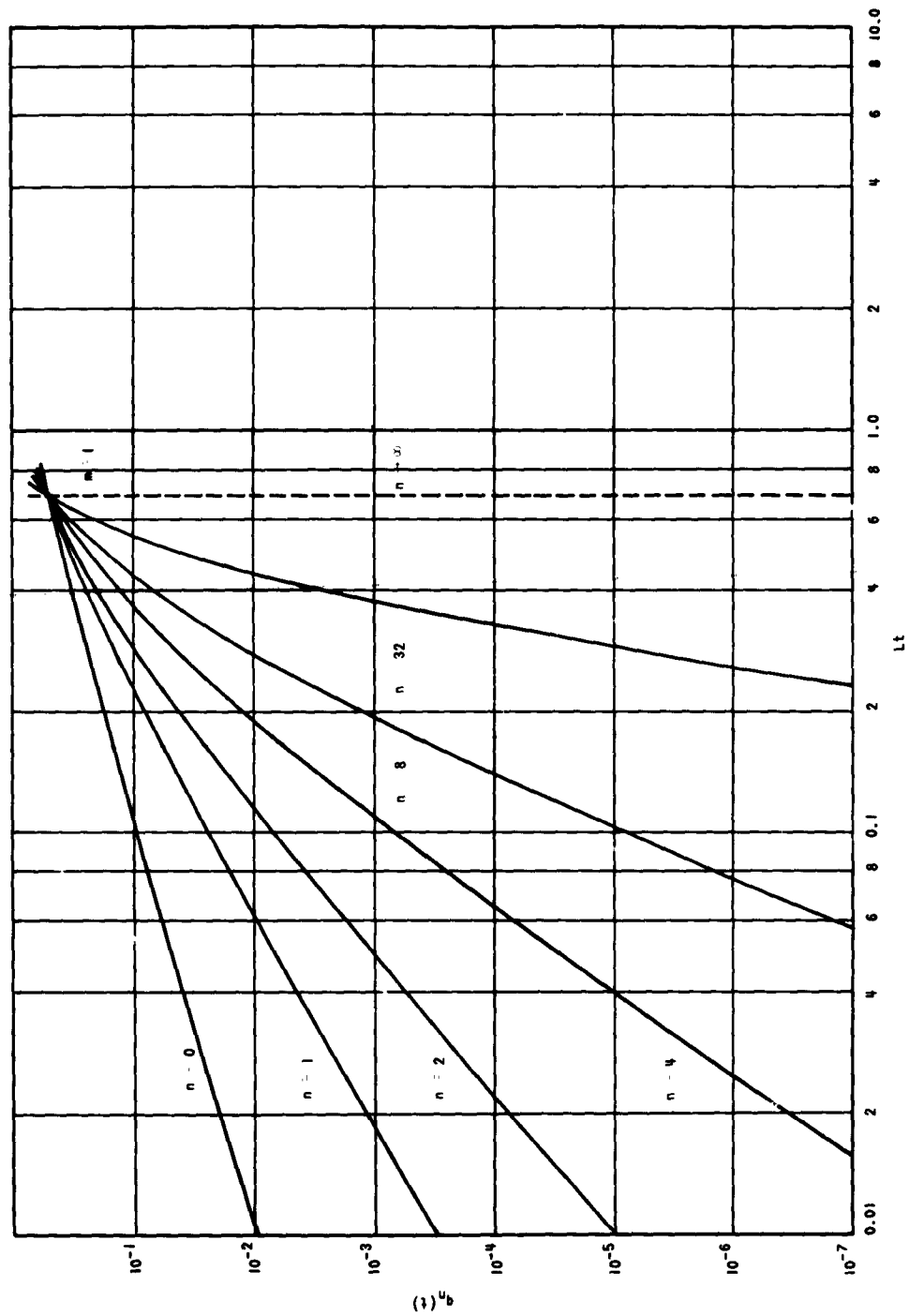


FIG. 2.4. PROBABILITY OF FAILURE $q_n(t)$ FOR A MAJORITY GROUP, SHOWN AS A FUNCTION OF Lt .

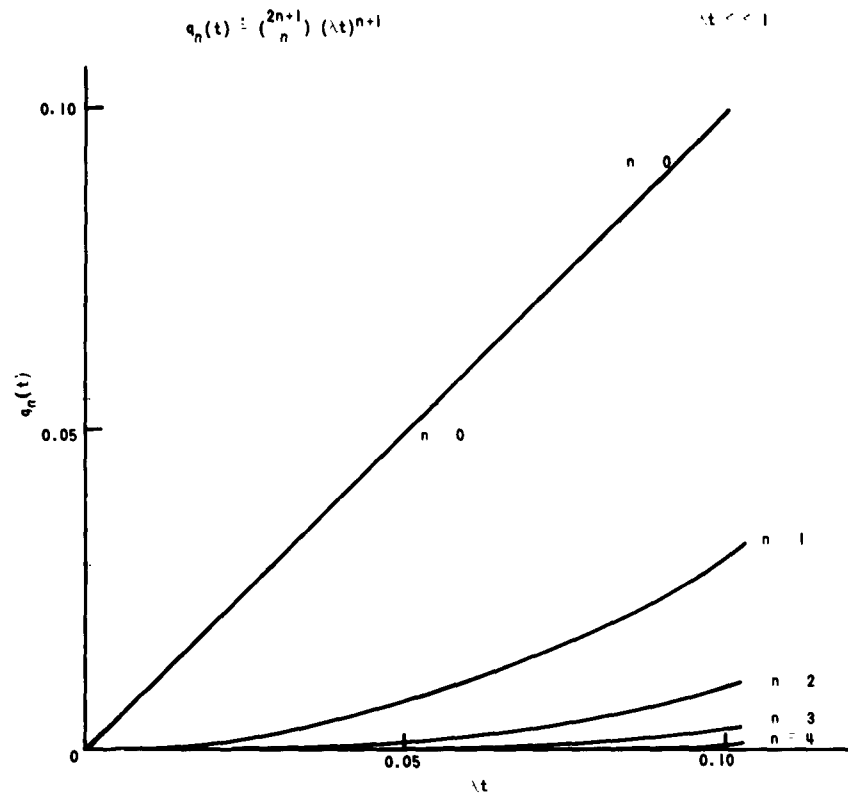


FIG. 2.5. PROBABILITY OF FAILURE $q_n(t)$ FOR A MAJORITY GROUP, SHOWN AS A FUNCTION OF t FOR $\lambda t \ll 1$.

of Fig. 2.6A. Let the failure rate for this system be L . If we think of this system as consisting of m (equal-sized) subsystems, then the failure rate of each subsystem will be

$$\lambda = \frac{L}{m} \quad (2.11)$$

Thus, if the system is divided into a large number of subsystems, then the failure rate of each subsystem will be much less than the failure rate of the overall system. Correspondingly the probability of survival for each subsystem will be considerably closer to one than the probability of survival for the overall system. Note that L , the failure rate for the overall nonredundant system, is fixed, whereas λ , the failure rate of the subsystem, depends on how small a portion of the system is considered a subsystem.

The probability of survival for the nonredundant subsystem is

$$p_o(t) = e^{-\lambda t} = e^{-(Lt/m)}$$

The probability of survival for the overall nonredundant system is

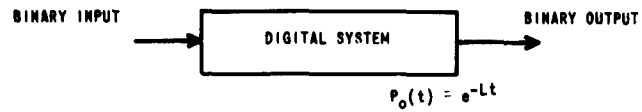
$$P_o(t) = [p_o(t)]^m = e^{-\lambda m t} = e^{-Lt} = 1 - Q_o(t) \quad (2.12)$$

Figure 2.6B shows a redundant system in which each of the m subsystems has been replaced by a majority group. The probability of survival for this system is, by (2.6):

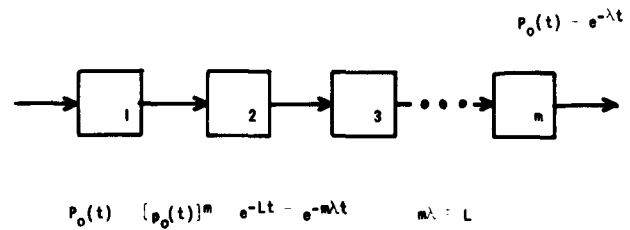
$$P_n(t) = [p_n(t)]^m = \left\{ \sum_{i=0}^n \binom{2n+1}{i} q_o^i p_o^{2n+1-i} \right\}^m = 1 - Q_n(t) \quad (2.13)$$

where

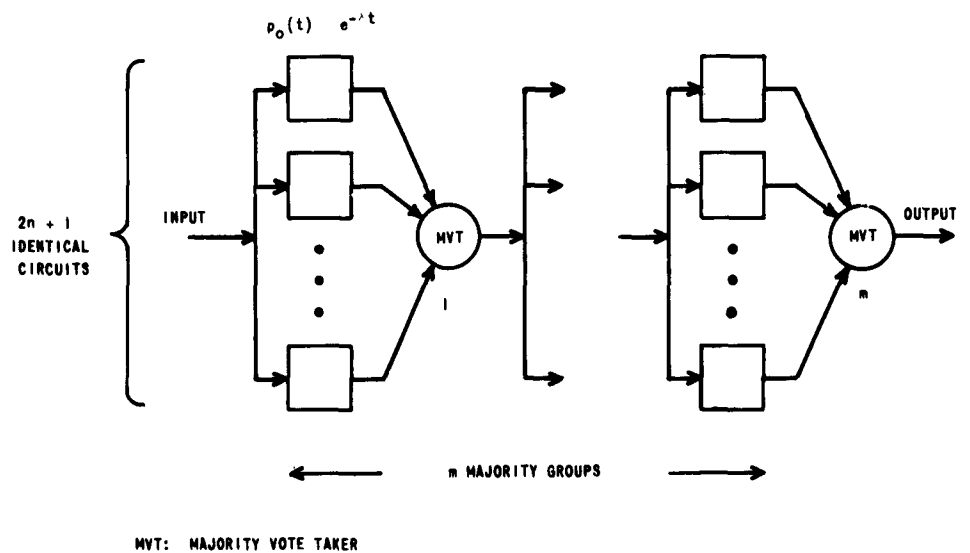
$$p_o = 1 - q_o = e^{-(Lt/m)}$$



EQUIVALENT REPRESENTATION OF THE ABOVE SYSTEM



A. Nonredundant digital system containing m subsystems



B. Redundant digital system containing m majority vote takers

FIG. 2.6. NONREDUNDANT SYSTEM AND CORRESPONDING REDUNDANT SYSTEM CONTAINING m MAJORITY VOTE TAKERS.

Figure 2.7 shows $P_n(t)$ as a function of t for a few values of n and m . The advantage of using a large number m of vote takers is readily apparent from these curves.

In the limiting case of $n \rightarrow \infty$ the MTF of the redundant system is

$$MTF \doteq 0.7 \frac{1}{\lambda} \doteq 0.7 \frac{m}{L} \lambda \quad (n \rightarrow \infty)$$

thus, in this limiting case the MTF increases proportionally with the number m of vote takers in the system.

Figure 2.8 shows $Q_n(t)$ vs Lt for a wide range of m and n . The fully drawn curves represent the exact form of $Q_n(t)$ as given by (2.13). The dotted curves represent the approximation for $Q_n(t)$ developed in (2.16) below.

Note that for $Q_n(t) \ll 1$ the steepness of the curves are determined by the amount of redundancy in the system and is virtually independent of the number of vote takers. The higher the redundancy, the steeper is the curve for $Q_n(t)$. On the other hand, increasing the number of restoring organs (vote takers) in the redundant system tends to move the curve for $Q_n(t)$ further to the right, thus improving the reliability of the system.

C. RELATIVE INCREASE IN T_{Δ} ACHIEVED BY THE USE OF REDUNDANCY AND PERFECT MAJORITY VOTE TAKERS

For many applications a computing system will only be useful during a period of time for which the probability of systems failure is much less than one. We therefore define the "useful life" T_{Δ} of a system to be that period of time for which a system can operate with the probability of failure being less than Δ where Δ is much smaller than one. (See Fig. 2.3). In this section we will derive a simple relationship between m , n , and $T_{\Delta n}$; and we will evaluate the improvement in reliability achieved by the use of redundancy and majority logic in terms of the ratio of $T_{\Delta n}$ for the redundant system to $T_{\Delta 0}$ for the nonredundant system.

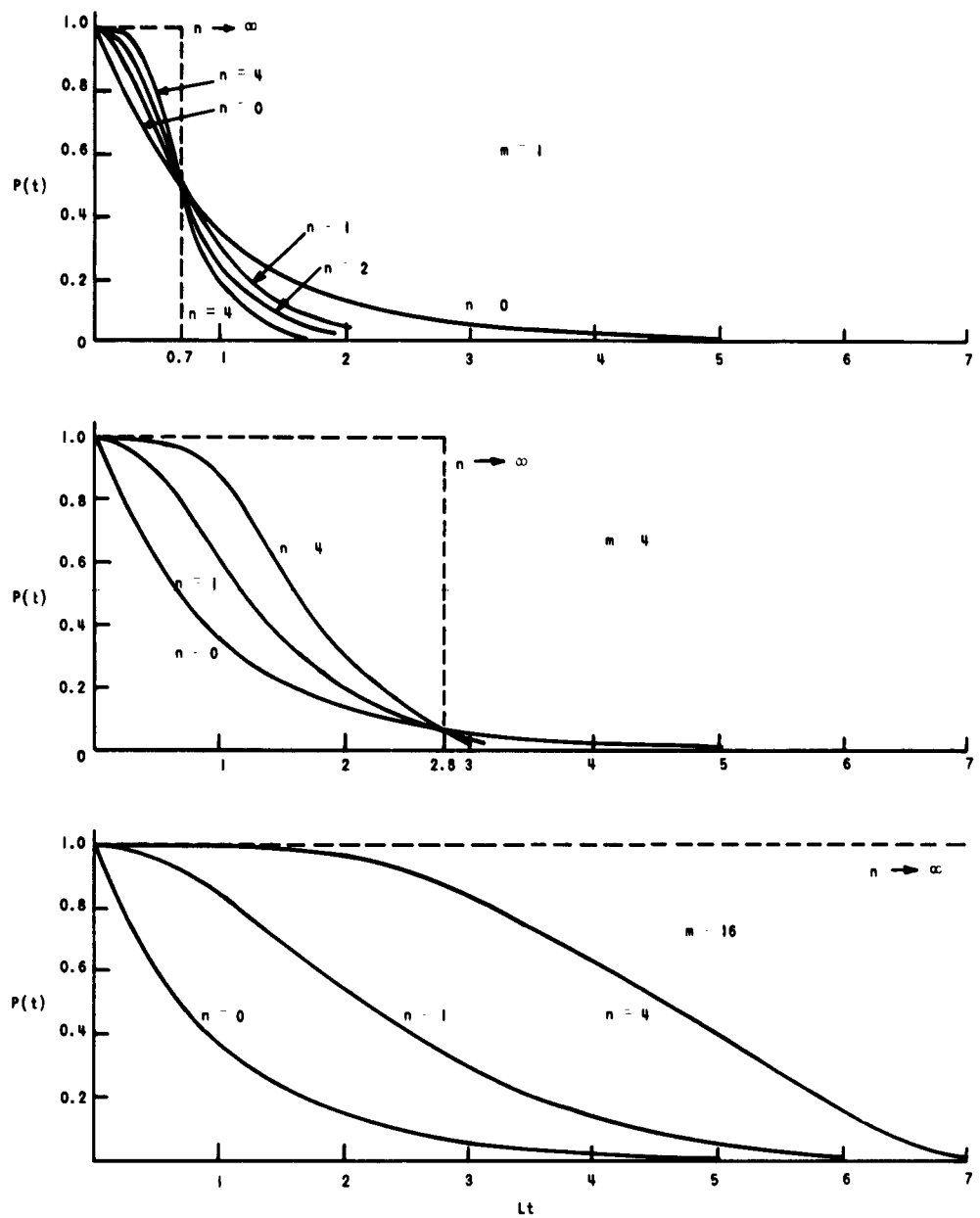
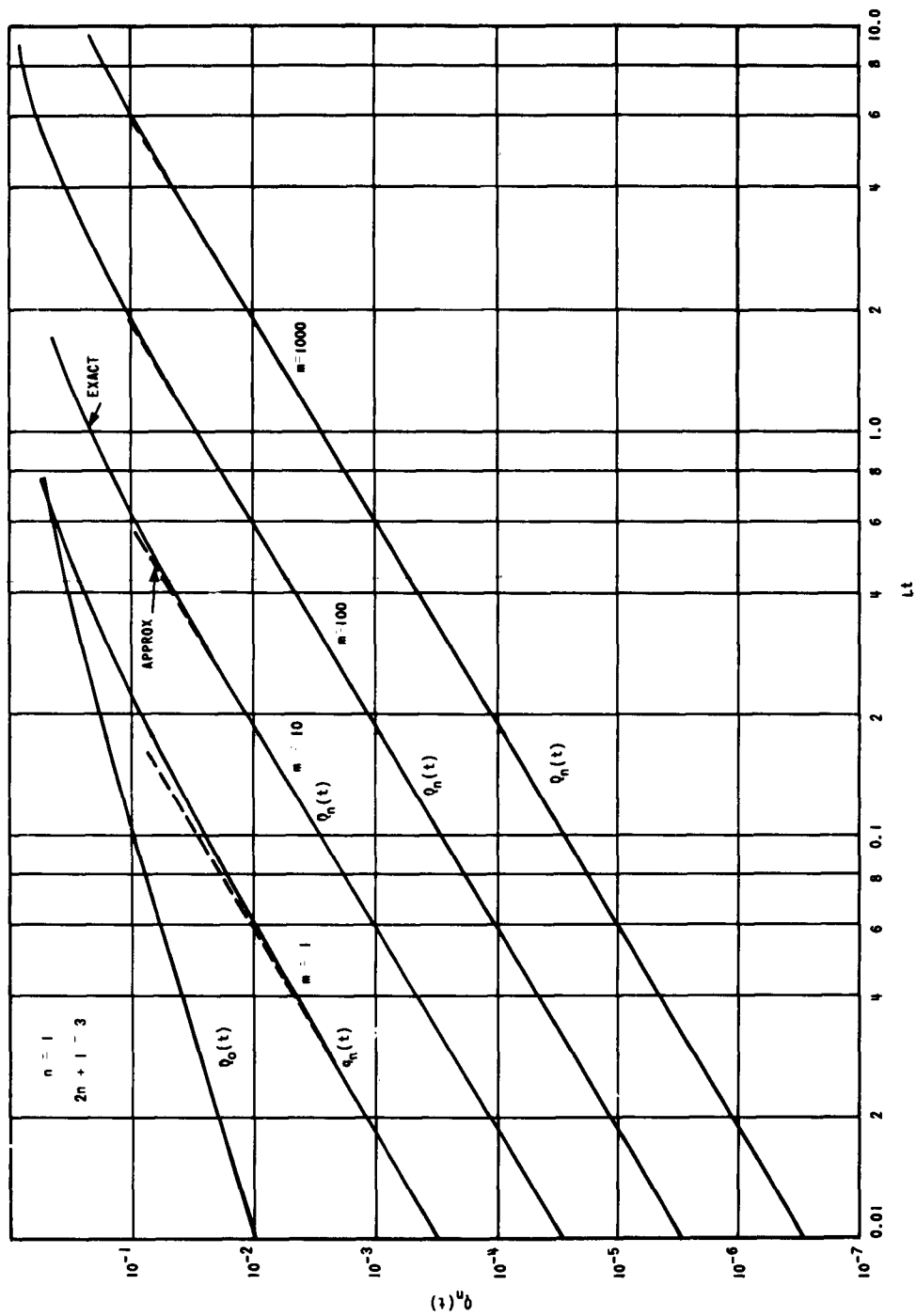
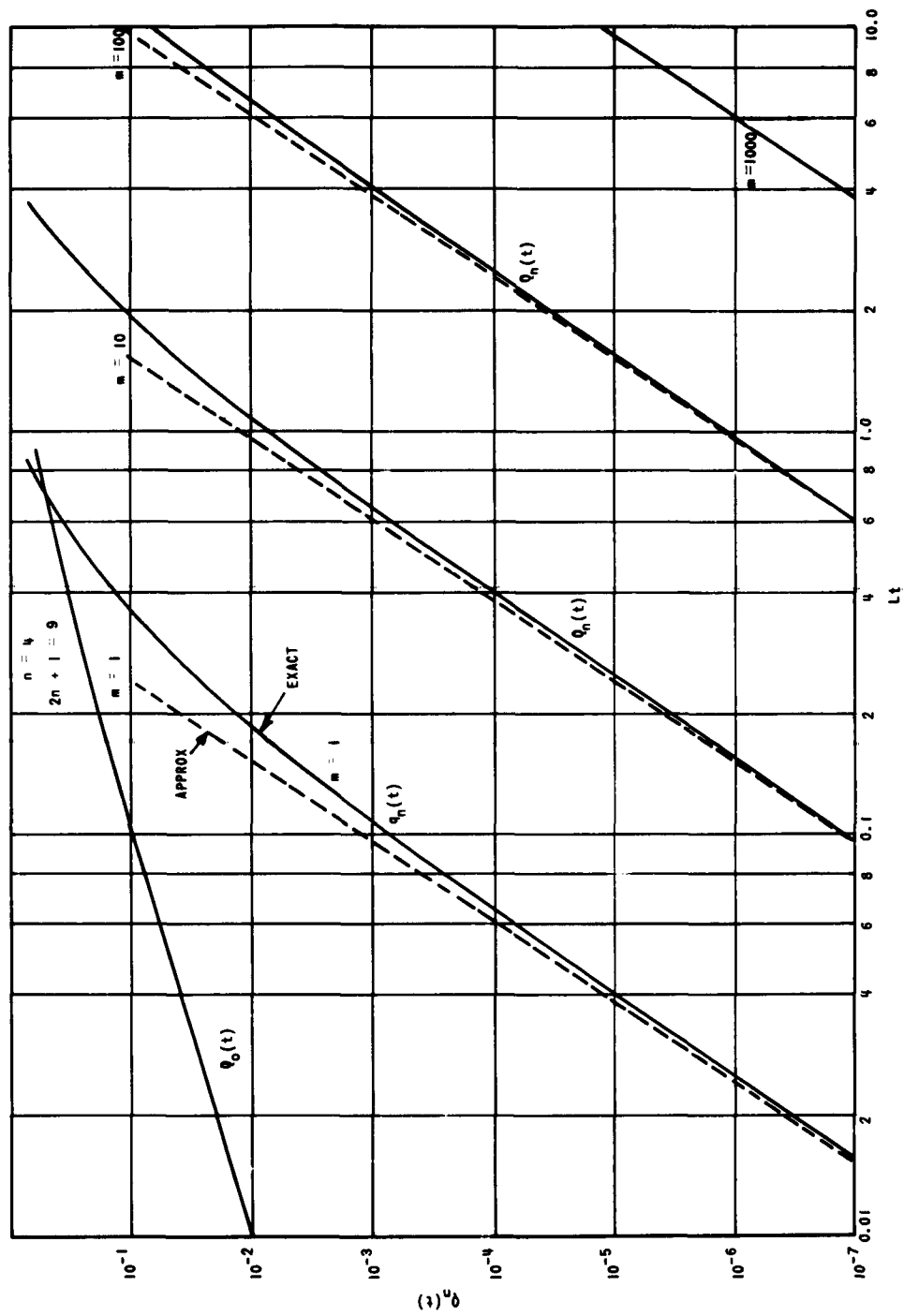


FIG. 2.7. PROBABILITY OF SURVIVAL $P_n(t)$ FOR A REDUNDANT DIGITAL SYSTEM USING m MAJORITY VOTE TAKERS.



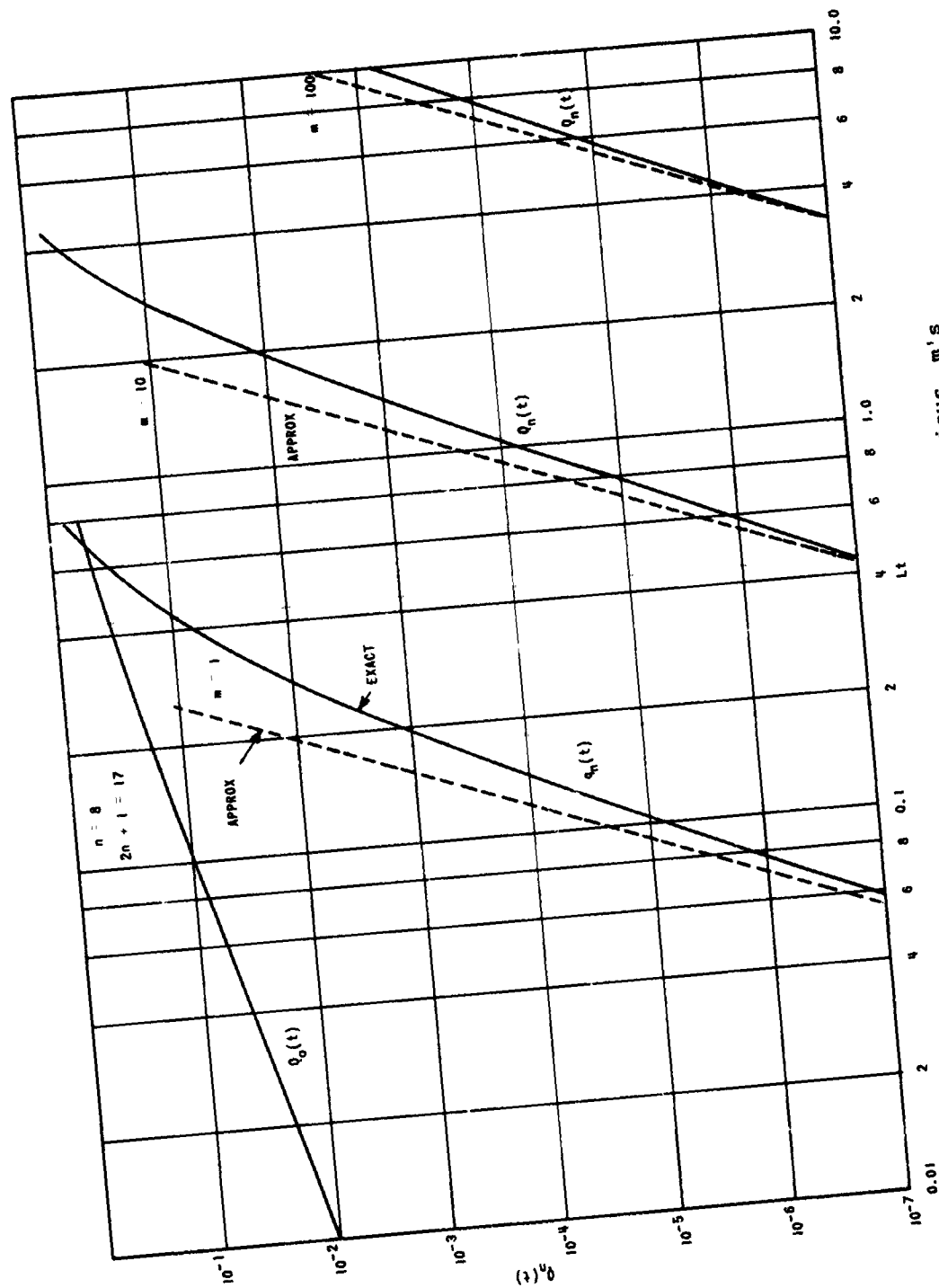
A. For $n = 1, 2n + 1 = 3$, and various m 's

FIG. 2.8. PROBABILITY OF FAILURE $Q_n(t)$ FOR A REDUNDANT DIGITAL SYSTEM CONTAINING m MAJORITY VOIE TAKERS.

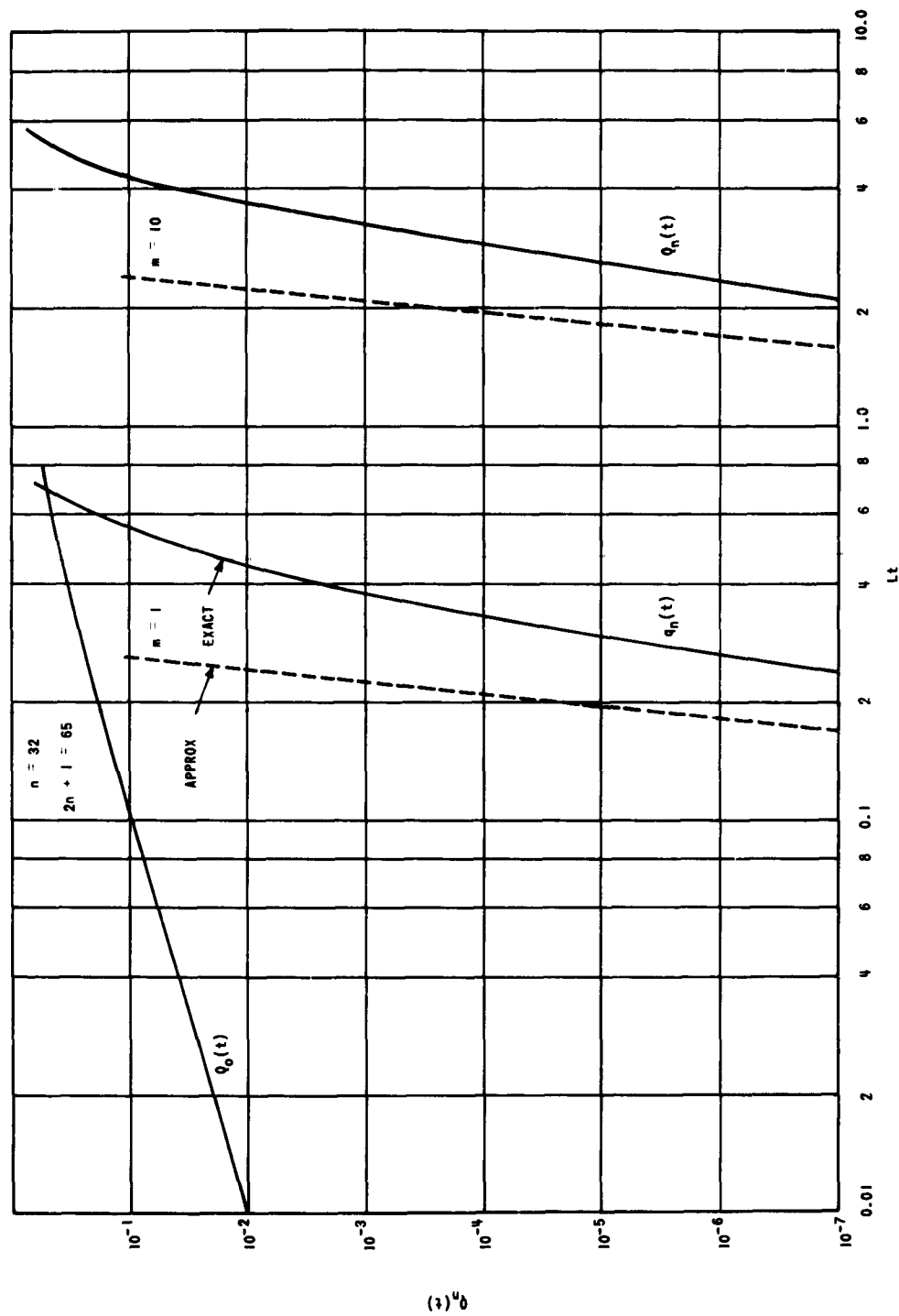


C. For $n = 4$, $2n + 1 = 9$, and various m 's

FIG. 2.8. PROBABILITY OF FAILURE $Q_n(t)$ FOR A REDUNDANT DIGITAL SYSTEM CONTAINING m MAJORITY VOTE TAKERS.

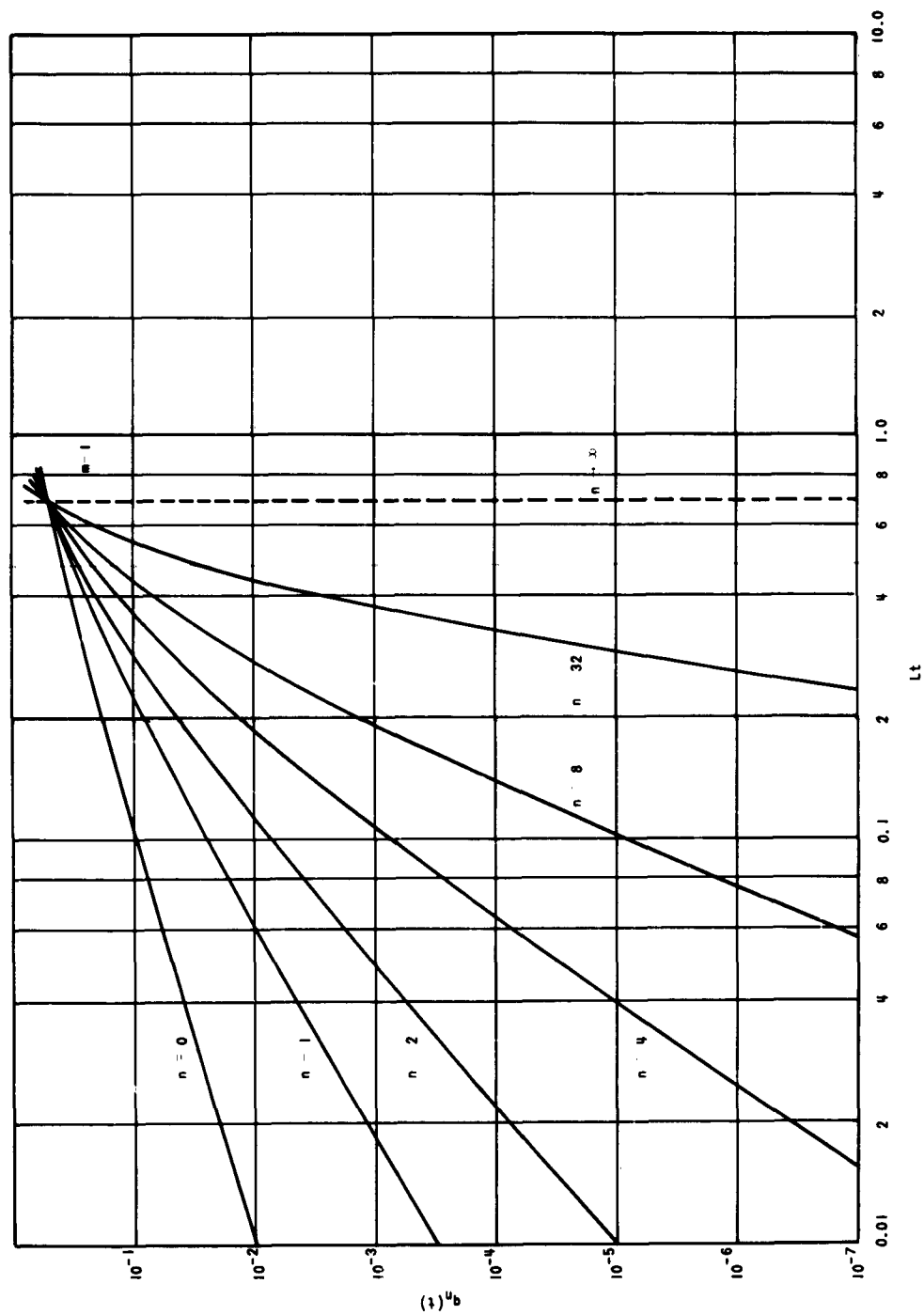


D. For $n = 8$, $2n + 1 = 17$, and various m 's
 FIG. 2.8. PROBABILITY OF FAILURE $Q_n(t)$ FOR A REDUNDANT DIGITAL SYSTEM CONTAINING m MAJORITY VOTE TAKERS.



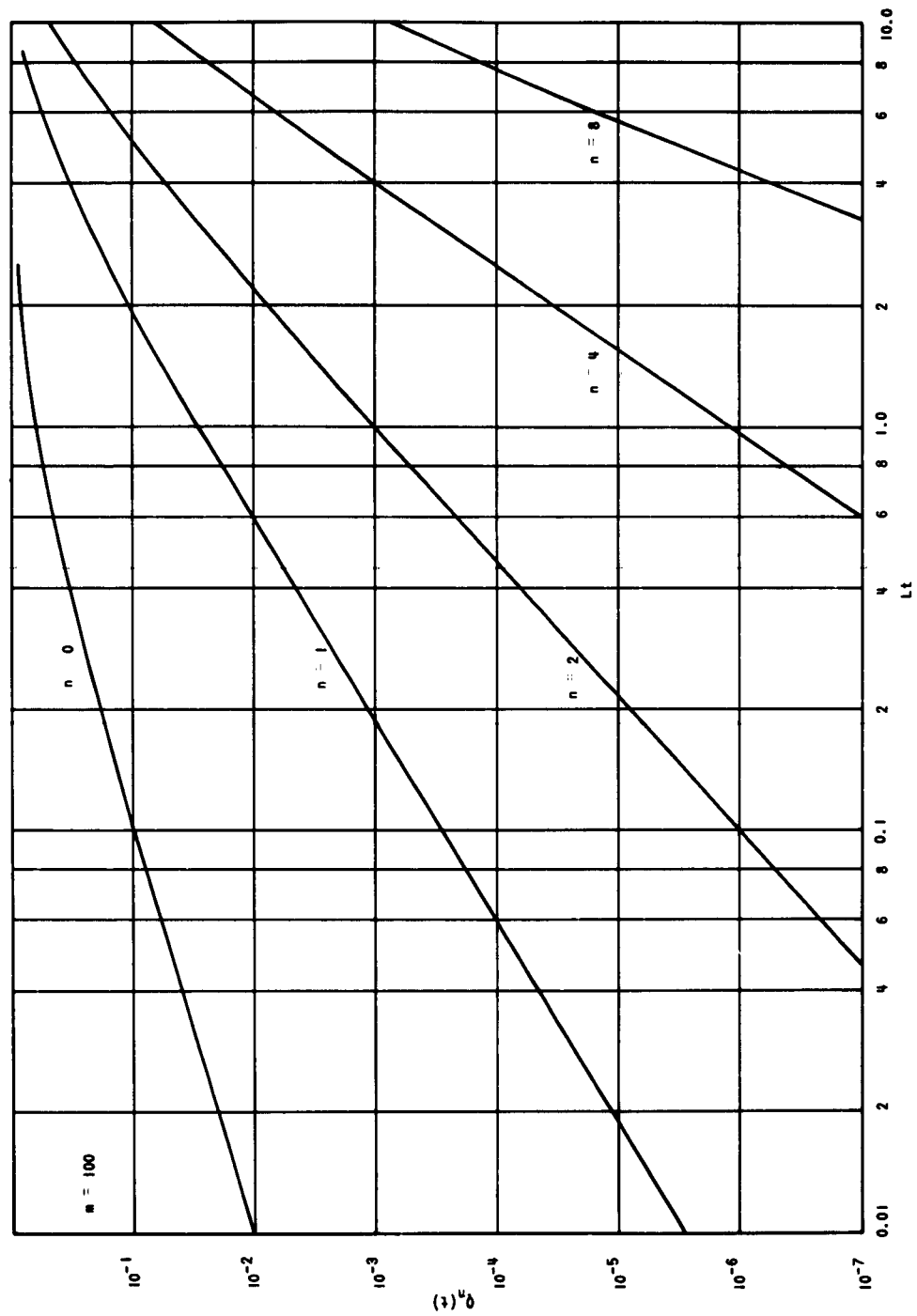
E. For $n = 32$, $2n + 1 = 65$, and various m 's

FIG. 2.8. PROBABILITY OF FAILURE $Q_n(t)$ FOR A REDUNDANT DIGITAL SYSTEM CONTAINING m MAJORITY VOTE TAKERS.



F. For $m = 1$ and various n 's

FIG. 2.8. PROBABILITY OF FAILURE $Q_n(t)$ FOR A REDUNDANT DIGITAL SYSTEM CONTAINING m MAJORITY VOTE TAKERS.



G. For $m = 100$ and various n 's

FIG. 2.8. PROBABILITY OF FAILURE $Q_n(t)$ FOR A REDUNDANT DIGITAL SYSTEM CONTAINING m MAJORITY VOTE TAKERS.

If $q_n(t)$ is the probability of failure for a single majority group, then the probability of failure for a system containing m majority groups is

$$Q_n(t) = 1 - [1 - q_n(t)]^m \quad (2.14)$$

from which it follows that

$$Q_n(t) \doteq mq_n(t) \quad \text{if} \quad mq_n(t) \ll 1 \quad (2.15)$$

inserting the expression (2.10) in (2.15) we next get

$$Q_n(t) \doteq m \binom{2n+1}{n} (\lambda t)^{n+1} \quad \text{if} \quad \begin{cases} \lambda t \ll 1 \\ mq_n(t) \ll 1 \end{cases} \quad \text{or if} \quad \lambda t \ll \left[\frac{1}{\binom{2n+1}{n}^m} \right]^{1/n+1} \quad (2.16)$$

and by equating $Q_n(t)$ with Δ

$$m \binom{2n+1}{n} (\lambda T_{\Delta n})^{n+1} \doteq \Delta \quad (2.17)$$

from which it follows that

$$T_{\Delta n} \doteq \frac{1}{\lambda} \left[\frac{\Delta}{\binom{2n+1}{n}} \right]^{1/n+1} \left[\frac{1}{m} \right]^{1/n+1} \quad (2.18)$$

under the conditions that

$$\lambda T_{\Delta n} \ll 1 \quad \text{as required by (2.10)} \quad (2.19)$$

and

$$\Delta \ll 1 \quad \text{as required by (2.15)} \quad (2.20)$$

The condition (2.19) can be written as

$$\left[\frac{\Delta}{\binom{2n+1}{n} m} \right]^{1/n+1} \ll 1 \quad (2.21)$$

next inserting $\lambda = L/m$ into (2.21)

$$T_{\Delta n} \doteq \frac{1}{L} \left[\frac{\Delta}{\binom{2n+1}{n} m} \right]^{1/n+1} m^{n/n+1} \quad \text{if} \quad \begin{cases} \Delta \ll 1 \\ \left[\frac{\Delta}{\binom{2n+1}{n} m} \right]^{1/n+1} \ll 1 \end{cases} \quad (2.22)$$

From (2.22) it is seen that when $n > 0$ then $T_{\Delta n}$ increases with the number m of vote takers, and if $n \gg 1$ then $T_{\Delta n}$ increases almost proportionally with m . Also, if $n \gg 1$ then $T_{\Delta n}$ does not depend very strongly on Δ when $\Delta \ll 1$. For the nonredundant system, $n = 0$,

$$T_{\Delta 0} \doteq \frac{1}{L} \Delta \quad \text{if} \quad \Delta \ll 1 \quad (2.23)$$

To evaluate the relative increase in useful life T_{Δ} achieved by the use of redundancy and majority logic consider the ratio

$$R_1 = \frac{T_{\Delta n}}{T_{\Delta 0}} \doteq \left[\frac{1}{\binom{2n+1}{n}} \right]^{1/n+1} \left(\frac{m}{\Delta} \right)^{n/n+1} \quad (2.24)$$

Note that R_1 increases "almost proportionally" with m for large n . Also note that R_1 is proportional with $(1/\Delta)^{n/n+1}$. This means that redundancy and majority logic is particularly effective in the case when we require that the probability of failure $Q(t)$ be very small during the mission time of the equipment; i.e., if we require $\Delta \ll 1$.

From the curves of Fig. 2.8 it is seen that the approximate expressions developed above lead to values of $T_{\Delta n}$ which tend to be smaller than the actual value of $T_{\Delta n}$. Thus, the improvement factor R_1 as given by (2.24) will be too small for large Δ . [The largest error results from approximating p^n by 1 when going from (2.3) to (2.8).]

In Table 1 is shown R_1 for various values of n and m/Δ .

TABLE 1. RELATIVE INCREASE R_1 IN "USEFUL LIFE" $T_{\Delta n}$ ACHIEVED BY THE USE OF REDUNDANCY AND MAJORITY LOGIC

$2n+1$	$\left[\frac{1}{\binom{2n+1}{n}}\right]^{1/n+1}$	R_1 for $\frac{m}{\Delta} = 10^2$	R_1 for $\frac{m}{\Delta} = 10^3$	R_1 for $\frac{m}{\Delta} = 10^4$	R_1 for $\frac{m}{\Delta} = 10^5$	$LT_{\Delta n}$ for $m=1$ $\Delta = 10^{-3}$	$LT_{\Delta n}$ for $m=100$ $\Delta = 10^{-3}$
1	-	1	1	1	1	0.001	0.001
3	0.58	5.8	18	58	180	0.018	0.18
5	0.46	10	46	215	1000	0.046	1.0
7	0.41	13	73	410	2300	0.073	2.3
9	0.38	15	95	600	3800	0.095	3.8
11	0.36	17	114	780	5300	0.114	5.3
13	0.35	18	129	930	6700	0.13	6.7
15	0.33	19	140	1060	7900	0.14	7.9
17	0.33	20	151	1170	9100	0.15	9.1
19	0.32	20	160	1270	10100	0.16	10.1
21	0.31	21	166	1360	11000	0.17	11
33	0.29	22	195	1700	14900	0.20	15
65	0.27	24	222	2080	19400	0.22	19

If the failure rate of the nonredundant system is known, then $T_{\Delta n}$ can be found from Table 1 by

$$T_{\Delta n} \doteq R_1 \frac{\Delta}{L}$$

D. RELATIVE INCREASE IN MTF ACHIEVED BY THE USE OF REDUNDANCY AND PERFECT MAJORITY VOTE TAKERS

An important property of a system using redundancy and restoring organs is the shape of the curve $P_n(t)$ versus time.

By differentiating $P_n(t)$ as given by (2.13) with respect to t we find that

$$\left. \frac{d}{dt} P_n(t) \right|_{t=0} = 0 \quad \text{when} \quad n > 0$$

$$\left. \frac{d}{dt} P_0(t) \right|_{t=0} = L \quad \text{when} \quad n = 0$$

Thus, initially, $P_0(t)$ will decrease at a rate L whereas $P_n(t)$ will have zero rate of decrease at $t=0$.

From Figs. 2.7 and 2.8 it is seen that for a highly redundant system, $n \gg 1$, $P_n(t)$ will be close to one or zero for most values of t . $T_{\Delta n}$ will therefore be a useful lower bound on the MTF for a system with high redundancy. (As noted in connection with (2.22) $T_{\Delta n}$ does not depend very strongly on Δ when $n \gg 1$.)

In Appendixes 1 and 2 we have developed somewhat more complicated expressions for a lower bound on the MTF for the redundant system. The expressions developed in the Appendixes are useful even in the case of low redundancy.

If we use $T_{\Delta n}$ as a lower bound on the MTF we find for the redundant system

$$MTF > \frac{1}{L} \left[\frac{\Delta}{\binom{2n+1}{n}} \right]^{1/n+1} m^{n/n+1} \quad \Delta \ll 1 \quad (2.27)$$

$$MTF > \frac{1}{L} K_1(\Delta, n) m^{n/n+1} \quad \Delta \ll 1 \quad (2.28)$$

where $1/L$ is the MTF for the nonredundant system and where

$$K_1(\Delta, n) = \left[\frac{\Delta}{\binom{2n+1}{n}} \right]^{1/n+1}$$

(2.27) and (2.28) are valid under conditions (2.20) and (2.21).

Using the results of Appendixes A and B we get

$$MTF > \frac{1}{L} K_2(n) \cdot m^{n/n+1} \quad (2.29)$$

where

$$K_2(n) = \left[\frac{3}{2} \frac{1}{\binom{2n+1}{n}} \right]^{1/n+1} \left(1 - \frac{3}{2} \frac{1}{n+2} + \frac{9}{8} \frac{1}{2n+3} - \frac{9}{16} \frac{1}{3n+4} \right) \text{ for } n > 1 \quad (2.30)$$

and

$$K_2(n) = 0.45 \quad \text{for } n = 1 \quad (2.31)$$

Table 2 gives values of $K_2(n)$ and $K_1(\Delta, n)$ for various values of n and Δ .

It is seen that (2.28) and (2.29) are fairly close for large values of n , whereas (2.29) is significantly better than (2.28) for small values of n .

From (2.29) it is seen that the relative increase in MTF obtained by the use of redundancy and majority logic is bounded below by

$$I(m, n) = K_2(n) \cdot m^{n/n+1} \quad (2.32)$$

Table 2 gives $I(m, n)$ for a few values of m and n .

TABLE 2. LOWER BOUND $I(m, n)$ ON THE RELATIVE INCREASE IN MTF OBTAINED BY THE USE OF REDUNDANCY AND MAJORITY LOGIC

$2n+1$	$K_1(\Delta, n)$ $\Delta = 1/10$	$K_1(\Delta, n)$ $\Delta = 1/100$	$K_2(n)$	$I(m, n)$ $m = 100$	$I(m, n)$ $m = 1000$
3	0.18	0.058	0.45	4.5	14
5	0.22	0.10	0.385	8.2	39
7	0.24	0.13	0.355	11	63
9	0.24	0.15	0.337	13	85
11	0.25	0.17	0.324	15	102
13	0.25	0.18	0.315	16	117
15	0.25	0.19	0.308	17	128
17	0.25	0.20	0.301	18	141
19	0.25	0.20	0.297	19	149
21	0.25	0.21	0.292	19	157
∞	0.69	0.69	0.69	69	690

E. CONCLUSIONS

Assuming ideal vote takers, the digital system will be most reliable if majority logic is applied at as low a level as possible, i.e., when the system is divided into as many digital subsystems, each followed by a majority vote taker, as possible.

On the other hand, it is clear that the MTF for the system will always be less than the MTF for the individual circuit. In the limit as $n \rightarrow \infty$ we have seen in Fig. 2.7 that the system MTF could be 0.69 times the MTF for the individual circuit. Equation (2.29) can be used to find a lower limit on the MTF for given values of m and n , if L is known.

Equation (2.32) gives a lower bound on the relative increase in MTF obtained by using redundancy and majority logic, even if λ and L are not known.

From Eqs. (2.22) and (2.32) it is seen that the use of redundancy and majority logic gives the greatest improvement in reliability in the case of large systems, i.e., in systems for which it is possible to achieve large values of m .

Finally, for a fixed mission time T , which is much shorter than the MTF, we have the following expressions for the probability of failure:

$$Q_0(T) = 1 - e^{-LT} \doteq LT \quad \text{if } LT \ll 1 \quad (2.12)$$

$$Q_n(T) \doteq m \binom{2n+1}{n} \left(\frac{L}{m} T \right)^{n+1} \quad \text{if } LT \ll \left[\frac{1}{\binom{2n+1}{n}} \right]^{1/n+1} m^{n/n+1} \quad (2.16)$$

so that

$$\frac{Q_0(T)}{Q_n(T)} = \frac{1}{\binom{2n+1}{n}} \left(\frac{m}{LT} \right)^n \quad \text{for } LT \ll 1 \quad (2.33)$$

(2.33) clearly shows the importance of making m the number of vote takers as large as possible.

It should be emphasized that the full improvement in reliability is realized only if we ensure that all circuits are working properly at time $t = 0$, that is, at the time when the mission is about to start.

Finally, it should be noted that it is possible to build a system in which the output from each circuit is compared with the output from the corresponding majority vote taker. If a circuit fails, it may then be possible to detect the failure and manually replace the circuit without interrupting the operation of the system.

III. MAJORITY LOGIC WITH NON-PERFECT VOTE TAKERS

Expressions for evaluating the improvement in reliability which can be achieved by the use of redundancy and majority logic were established in Chapter II. The analysis of Chapter II was based on the use of perfect infallible vote takers.

In this chapter the case of non-perfect vote takers is considered. We shall first determine a condition on the failure rate λ_v of the vote takers in order that the vote takers can be considered ideal (Eq. 3.7).

If the vote takers can not be considered ideal, the use of redundant vote takers is recommended. By appropriately modifying the expressions of Chapter II to include the effect of the unreliable vote takers, we next established for the redundant system a lower bound T_1 on the MTF and an approximation for $T_{\Delta n}$ as a function of redundancy, number of vote takers, vote taker reliability, and system reliability for the nonredundant system.

Finally, we established an expression for the optimum number of redundant unreliable vote takers to be used in the system.

A. CONDITIONS FOR CONSIDERING THE VOTE TAKERS TO BE IDEAL

In the following we shall establish a condition on λ_v , the failure rate of the majority vote taker, in order that the MVT can be considered ideal. Let p_v be the probability that a vote taker is working properly, then $q_v = 1 - p_v$ will be the probability that the vote taker is not working properly. We will assume that if the vote taker is not working properly, then it will have as an output the complement of the desired output. (The implications of this assumption are discussed in Chapter IIA.)

We will assume that the number of vote-taker failures in a given length of time obeys the Poisson distribution; furthermore, let the MTF of a vote taker be $1/\lambda_v$. Then

$$p_v(t) = e^{-\lambda_v t} \quad (3.1)$$

and the probability that m vote takers are all working properly becomes

$$P_v(t) = [p_v(t)]^m = e^{-\lambda_v m t} \quad (3.2)$$

We will establish a condition on the failure rate λ_v of the vote takers, such that the probability of failure for the system shown in Fig. 2.6B is essentially unaffected by the failures of the vote taker, that is, we will establish the condition on λ_v for the vote takers to be considered ideal.

Consider the system of Fig. 2.6B. The probability of failure for the overall system assuming ideal vote takers is

$$Q_n(t) \leq \Delta \quad \text{for } t \leq T_\Delta$$

The probability that none of the vote takers will fail is

$$P_v(t) = e^{-m\lambda_v t} \quad (3.3)$$

and the probability of failure among the vote takers is

$$Q_v(t) = (1 - e^{-m\lambda_v T_\Delta}) \quad \text{for } t = T_\Delta \quad (3.4)$$

Thus, if

$$Q_v(t) \ll \Delta \quad \text{for } t \leq T_\Delta \quad (3.5)$$

then the vote taker can be considered to be ideal. (3.5) is satisfied if

$$m\lambda_v T_\Delta \ll \Delta \quad (3.6)$$

or

$$\lambda_v \ll \frac{\Delta}{m} \frac{1}{T_\Delta} \doteq \frac{\Delta}{m} L \left[\frac{\binom{2n+1}{n}}{\Delta m^n} \right]^{1/n+1} \quad (3.7)$$

Hence, the vote takers can be considered ideal if the mean time between failures

$$MTF_v = \frac{1}{\lambda_v} \gg \frac{1}{L} \left[\frac{m^{2n+1}}{\Delta^n \binom{2n+1}{n}} \right]^{1/n+1} \quad (3.8)$$

Note that the requirement to the reliability of the vote taker increases as Δ is decreased. Fig. 3.1 indicates the reason for this. Recall that

$$\frac{dP_n(t)}{dt} = 0 \quad \text{at} \quad t=0$$

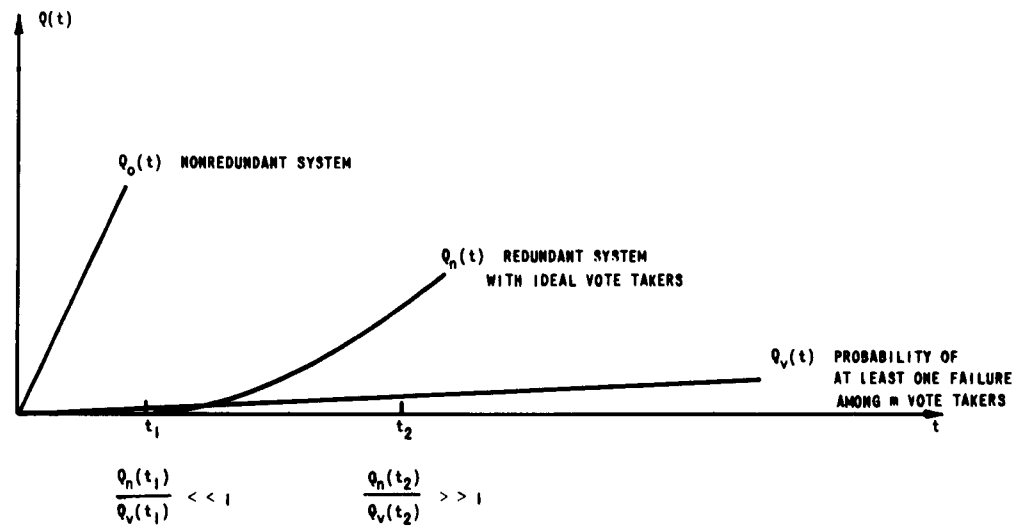


FIG. 3.1. PROBABILITY OF AT LEAST ONE FAILURE AMONG m VOTE TAKERS COMPARED WITH THE PROBABILITY OF FAILURE AMONG m MAJORITY GROUPS.

for the redundant system with ideal vote taker. Also note that for the nonredundant vote takers

$$\frac{dP_v(t)}{dt} = -m\lambda_v \quad \text{at } t=0$$

Hence, for any $\lambda_v > 0$ there will always be some period of time for which the (nonredundant) vote takers will be the major contributor to the system failures.

B. USEFUL LIFE T_{Δ} AND MTF ACHIEVED BY REDUNDANCY AND NON-PERFECT VOTE TAKERS

In the case when the failure rate of the vote takers cannot be neglected, we can use redundant vote takers as shown in Fig. 3.2A. The system of Fig. 3.2A can be represented as shown in Fig. 3.2B as a system using nonredundant ideal vote takers in which the failure rate of the individual circuit is the sum of the failure rate of the original circuit and the failure rate of the vote taker feeding that circuit. The probability of survival for the individual circuit of Fig. 3.2B is

$$p = p_v p_o = e^{-\lambda_v t} e^{-\lambda_o t} = e^{-(\lambda_v + \lambda_o)t} \quad (3.9)$$

or using $\lambda_o = \left(\frac{1}{m}\right)L$

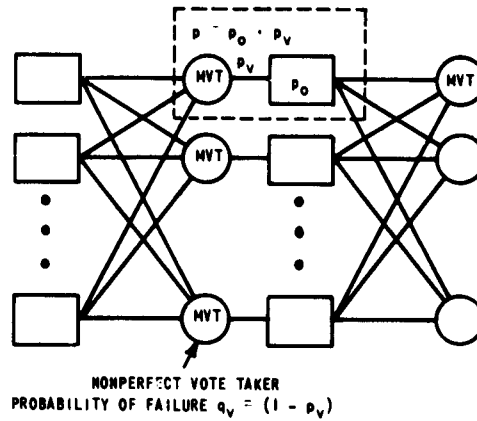
$$p = e^{-\left(\frac{L}{m} + \lambda_v\right)t} \quad (3.10)$$

Then by (2.13) we get

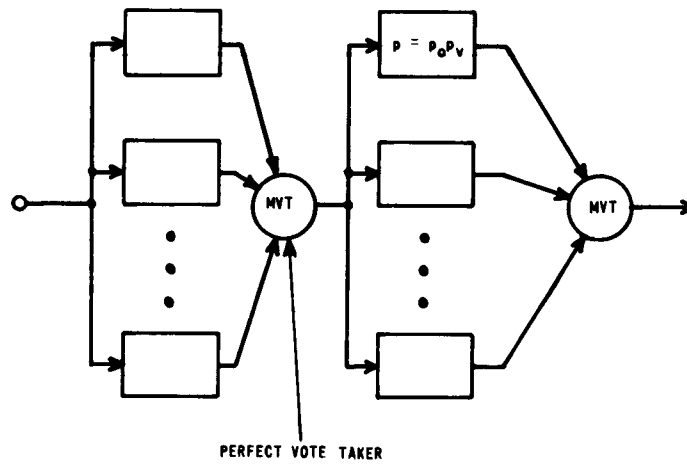
$$P_n(t) = \left\{ \sum_{i=0}^n \binom{2n+1}{i} q^i p^{2n+1-i} \right\}^m = 1 - Q_n(t)$$

where

$$p_0 = e^{-\lambda t}$$



A. System containing redundant majority vote takers



B. Model for analyzing system containing redundant majority vote takers

FIG. 3.2. USE OF REDUNDANT MAJORITY VOTE TAKERS.

$$p = 1 - q = \exp \left\{ -\frac{L}{m} + \lambda_v \right\} = \exp \left\{ -\frac{L + m\lambda_v}{m} t \right\}$$

The curves of Figs. 2.7 and 2.8 show respectively the probability of survival $P_n(t)$ and the probability of failure $Q_n(t)$ as a function of Lt for a redundant system using perfect vote takers. Note that the same curves apply to a redundant system using redundant non-perfect vote takers if Lt on the abscissa is replaced by $(L + m\lambda_v)t$. (This change in scale on the abscissa clearly does not apply to the curve for the nonredundant system.)

To find $T_{\Delta n}$ for the system of Fig. 3.2 substitute $(\frac{L}{m} + \lambda_v)$ for λ in Eq. (2.18) and get

$$T_{\Delta n} = \frac{1}{\frac{L}{m} + \lambda_v} \left[\frac{\Delta}{\binom{2n+1}{n}_m} \right]^{1/n+1} \quad (3.11)$$

or

$$T_{\Delta n} = \frac{m^{n/n+1}}{L + m\lambda_v} \left[\frac{\Delta}{\binom{2n+1}{n}} \right]^{1/n+1} \quad (3.12)$$

Similarly, substitute $(L + m\lambda_v)$ for L in (2.29) to get

$$MTF > \frac{1}{L + m\lambda_v} \cdot m^{n/n+1} \cdot K_2(n) = T_1 \quad (3.13)$$

C. OPTIMUM NUMBER OF REDUNDANT NON-PERFECT VOTE TAKERS

In the case of the ideal vote taker, $T_{\Delta n}$ and T_1 would increase proportionally with $m^{n/n+1}$; thus, for a given redundancy $T_{\Delta n}$ and T_1 would be maximized if vote takers are applied at the lowest possible level (i.e., if m is made as large as possible). Alternatively, m and thereby T_{Δ} might be limited by the cost of the vote takers.

If the vote taker is unreliable, then the reliability of the system will actually start to decrease if the number of vote takers is increased above some optimum value.

To find the optimum value of m , for a fixed n , differentiate (3.12) and/or (3.13) and find

$$\frac{d}{dm} \left\{ \frac{m^{n/n+1}}{L + m\lambda_v} \right\} = 0 \quad \text{when } m = \frac{Ln}{\lambda_v} \quad n \geq 1$$

thus, the maximum of $T_{\Delta n}$ and T_1 is achieved for

$$m = \frac{Ln}{\lambda_v} = M \quad (3.14)$$

Inserting $M\lambda_o = L$ in (3.14) we find

$$M\lambda_v = Ln = M\lambda_o n \quad (3.15)$$

thus maximum reliability is achieved if:

$$\frac{\lambda_o}{\lambda_v} = \frac{1}{n} \quad (3.16)$$

For $n = 1$, i.e., 3 circuits in parallel, the optimum division of the system will then be such that the failure rate λ_o of the individual circuit is equal to the failure rate λ_v of the vote taker. For larger values of n the optimum division will be such that the failure rate of the individual circuit is actually less than the failure rate of the vote taker.

Clearly, the optimum value of m as found in (3.14) will also minimize $Q_n(t)$. This can be checked by replacing λ in Eq. (2.16) by $(\frac{L}{m} + \lambda_v)$ and differentiating with respect to m .

Equation (3.14) gives a value of m which should not be exceeded. Several reasons might exist why a smaller value of m will be used in any given situation. One obvious constraint on m is the cost of the vote takers.

In most practical cases systems complexity or systems cost will present a constraining factor. Increases in n and m each represent an increase in systems cost (or complexity). For a given situation once the relative cost of increasing n or m has been established, Eq. (3.12) or (3.13) can be used to establish the trade-off between n and m . It is seen from (3.12) that when $m \ll M$, then $T_{\Delta n}$ will increase "almost proportionally" with m , whereas when m gets closer to M , then $T_{\Delta n}$ will increase much more slowly as a function of m .

Furthermore, the smallest block to which majority logic can be applied must itself be a digital unit. Since a majority vote taker is a rather simple circuit, it will in most practical situations not be possible to achieve the optimum value of m , since the smallest digital unit in the system will usually have a failure rate larger than the failure rate of the vote taker.

IV. MAJORITY LOGIC WITH ELIMINATION OF UNRELIABLE CIRCUITS - ADAPTIVE VOTE TAKERS

In this chapter we consider the use of adaptive vote takers (AVT) as the restoring organ in a redundant system. An expression for the probability of failure $Q_{na}(t)$ for a redundant system using AVT's is established; and we compare graphically the reliability which can be achieved in a redundant system by the use of AVT's with the reliability which can be achieved by the use of Majority Vote Takers (MVT). We find that under a wide range of conditions the use of approximately 10 MVT's in place of each AVT will, for a fixed amount of redundancy, result in a system of superior reliability.

A. ADAPTIVE VOTE TAKERS VERSUS MAJORITY VOTE TAKERS

For a fixed amount of redundancy greater than 3 and a fixed number of vote takers the reliability of a system can be further improved, if the majority vote taker is replaced by an adaptive vote taker. By comparing the output from each individual circuit in a majority group with the output from the vote taker, it is possible to estimate the error rate of the individual circuits. An optimum voting procedure can then be established in which the more reliable circuits carry more weight in the voting than do the less reliable circuits. Pierce [Ref. 7] has established the voting procedure which will minimize the probability of error in the output of the vote taker for arbitrary known error rates of the individual circuits. This optimum voting procedure requires that the vote weight of the individual circuits can be set to any value between 1 and 0 depending on the error rate of the circuit.

In its simplest (non-optimum) form, an adaptive vote taker gives either weight one or weight zero to a circuit; i.e., initially all circuits carry weight 1 until the error rate of one circuit increases beyond some threshold level, in which case that circuit is eliminated from the vote taker (see Fig. 4.1). The adaptive vote taker will thus eliminate the circuits as they fail and the "adaptive majority group" may then operate as long as at least two circuits in the group are operating properly.

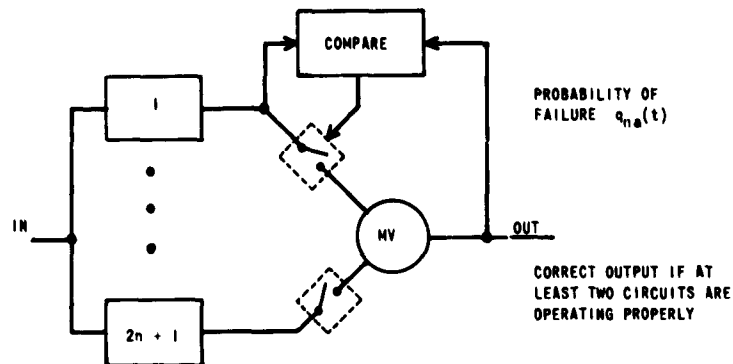


FIG. 4.1. ADAPTIVE MAJORITY GROUP. (Majority vote with elimination.)

We shall restrict our discussion in the following to this type of adaptive vote taker only.*

If the probability of failure for the individual circuit is

$$q_o(t) = (1 - e^{-\lambda t})$$

and if the adaptive vote taker is perfect, then the probability of failure $q_{na}(t)$ for the adaptive majority group is

$$q_{na}(t) = q_o^{2n+1} + (2n+1)p_o \cdot q_o^{2n} \quad (4.1)$$

* In many practical situations a circuit will either be working almost perfectly, i.e., with an error rate which is many orders of magnitude less than one, or it will have failed completely, i.e., the output from the circuit will be independent of the input. If the circuits under consideration are indeed in one or the other of the above two states (most of the time), then the reliability which can be achieved by using an adaptive vote taker, which eliminates the failed circuits, will be (virtually) as good as the reliability which can be achieved by using an adaptive vote taker which has continuous weights.

Figure 4.3 shows $q_{na}(t)$ as a function of t for n ranging from 2 to 32. (The curves marked by A). For comparison the probability of failure $q_n(t)$ for the corresponding majority group is also shown in Fig. 4.3.

Over the range of t for which $q_n(t) \ll 1$ we find as expected that $q_{na}(t) \ll q_n(t)$; thus, as a restoring organ, the adaptive vote taker is superior to the majority vote taker.

It appears that an adaptive vote taker will be considerably more difficult to realize than a majority vote taker. It may therefore be more reasonable to compare the reliability which is achieved by using one adaptive vote taker with the reliability which could be achieved by using m majority vote takers in the same system (see Fig. 4.2).

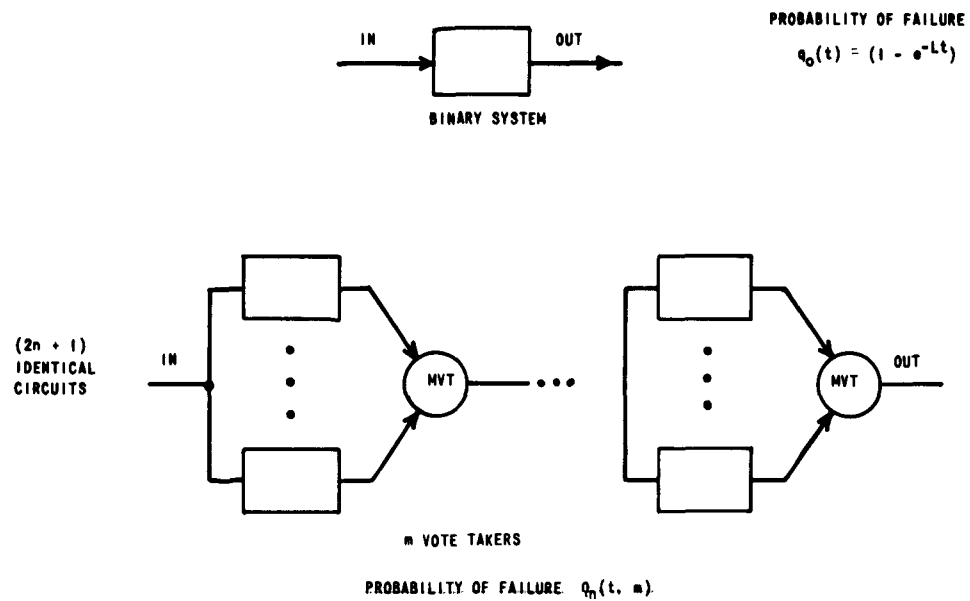
In Fig. 4.3 we have shown the probability of failure $Q_n(t, m)$ versus time for a system using m (ideal) majority vote takers, for $m = 1, 5, 10$, and 20 . Also shown is the probability of failure for the same system using one adaptive vote taker. First, a remark about the behaviour of $Q_n(t, m)$ and $q_{na}(t)$ for t close to zero; from (2.16) by replacing λ by L/m

$$Q_n(t, m) \doteq \frac{1}{m^n} \binom{2n+1}{n} (Lt)^{n+1} \quad \text{for } Lt \ll 1 \quad (4.2)$$

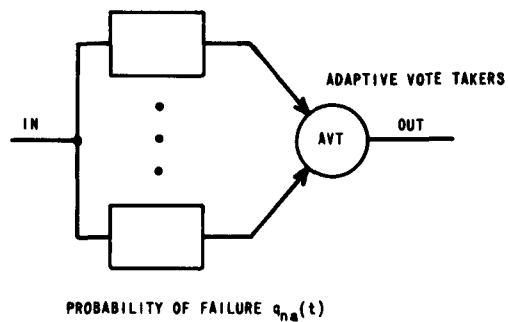
from (4.1)

$$q_{na}(t) \doteq (2n+1) (Lt)^{2n} \quad \text{for } Lt \ll 1 \quad (4.3)$$

Since $q_{na}(t)$ goes as t to the power $2n$, whereas $Q_n(t)$ goes as t to the power $(n+1)$, it is clear that regardless of the value of m there will always be a range of t such that $Q_n(t) > q_{na}(t)$ for $0 \leq t \leq t_1$. It is interesting to compare the behaviour of $q_{na}(t)$ with the behaviour of $Q_n(t)$ for $m = 10$. From the curves of Fig. 4.3 we see that (for $2 \leq n \leq 32$) $Q_n(t)$ and $q_{na}(t)$ have a cross-over point, and that $Q_n(t)$ actually is significantly smaller than $q_{na}(t)$ over a considerable range of t and q . For $n = 2$ and $m = 10$ the cross-over is seen to take place at $q \sim 10^{-6}$. For $n = 8$ and $m = 10$ the cross-over takes place at $q \sim 10^{-8}$. As n increases, the cross-over takes

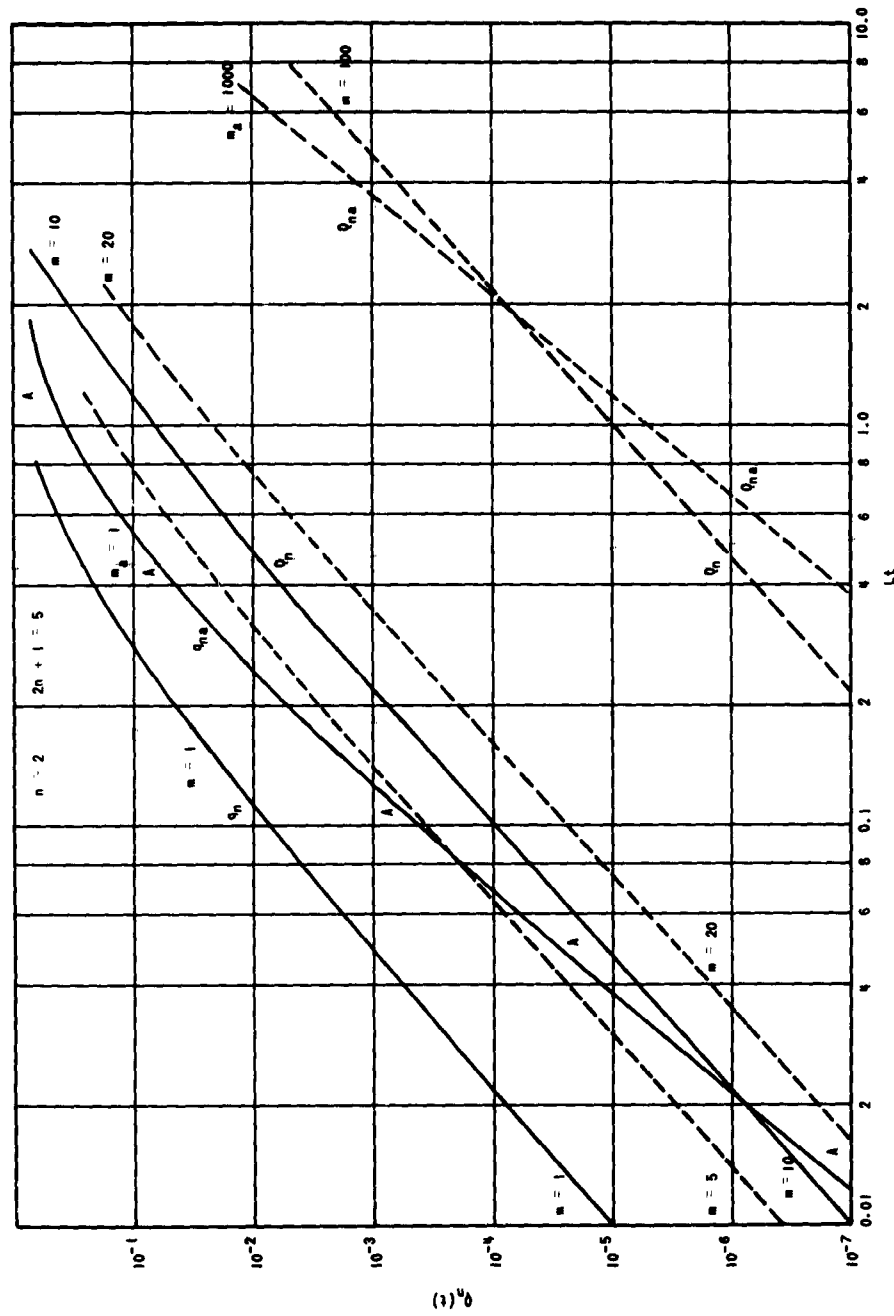


A. System using m majority vote takers



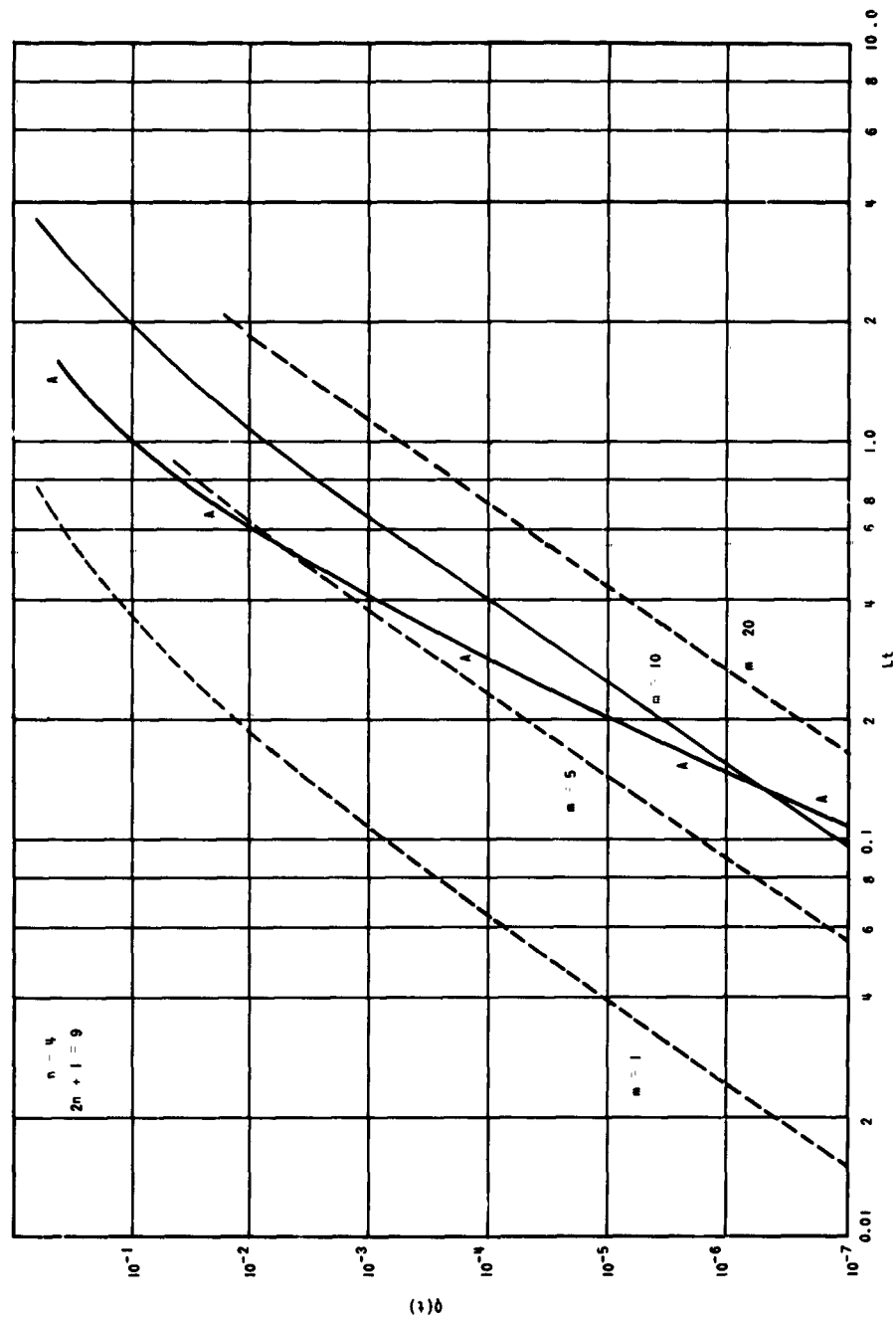
B. System using one adaptive vote taker

FIG. 4.2. COMPARISON BETWEEN A REDUNDANT SYSTEM USING ONE ADAPTIVE VOTE TAKER AND A REDUNDANT SYSTEM USING m MAJORITY VOTE TAKERS.



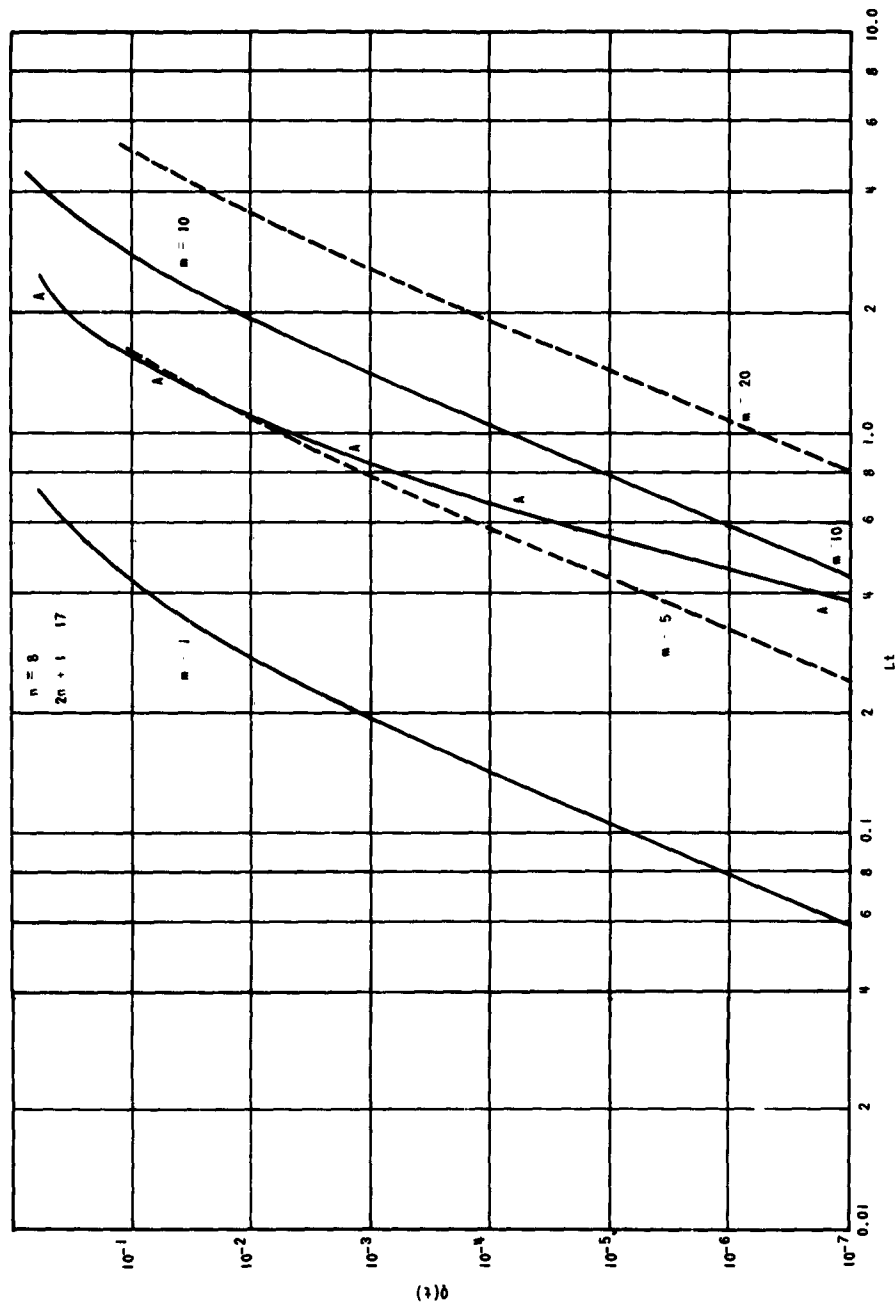
A. For $n = 2$, $2n + 1 = 5$, and m_a adaptive vote takers

FIG. 4.3. PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM CONTAINING ADAPTIVE VOTE TAKERS COMPARED WITH THE PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM USING m MAJORITY VOTE TAKERS.



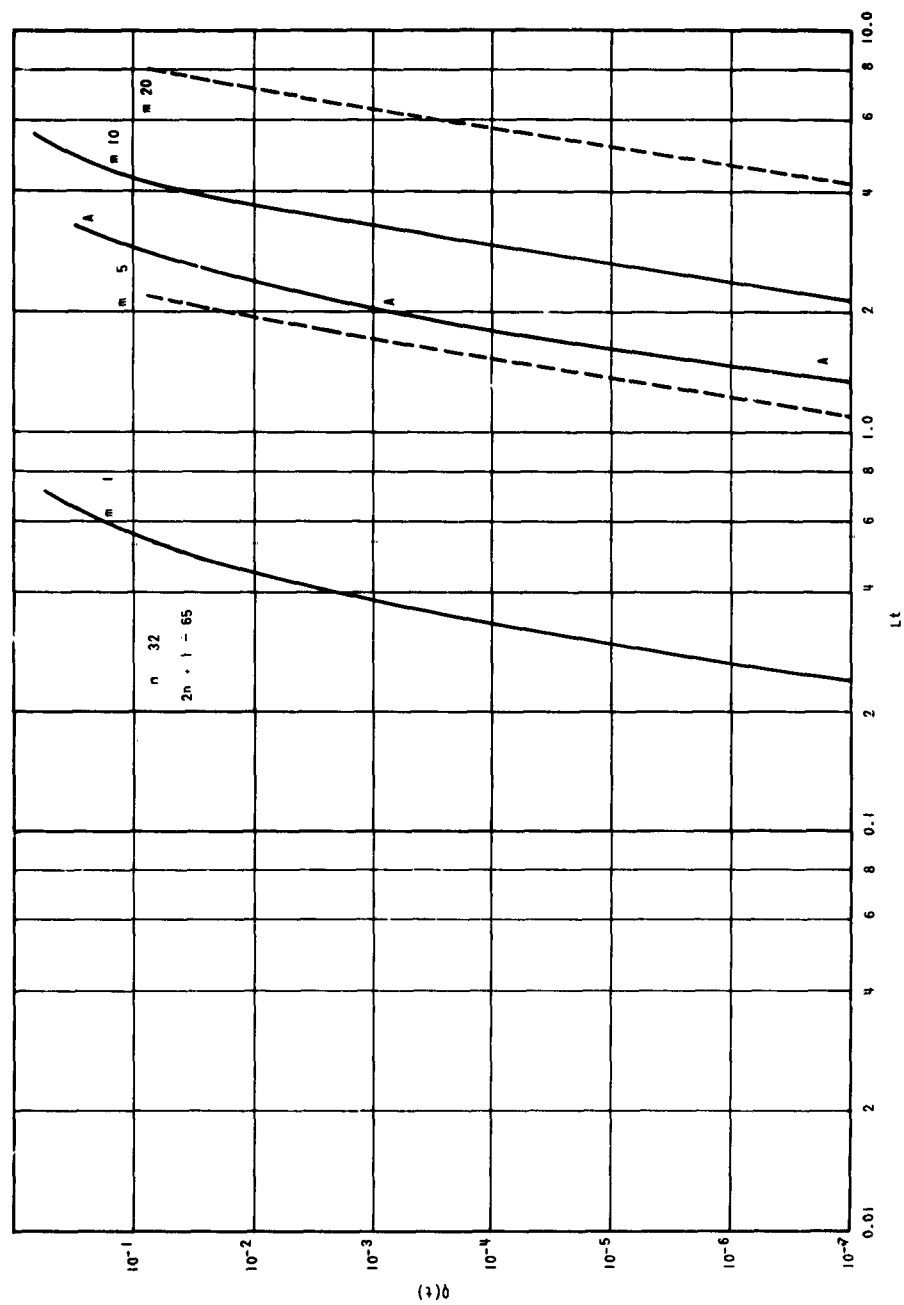
B. For $n = 4$, $2n + 1 = 9$, and one adaptive vote taker

FIG. 4.3. PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM CONTAINING ADAPTIVE VOTE TAKERS COMPARED WITH THE PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM USING m MAJORITY VOTE TAKERS.



C. For $n = 8$, $2n + 1 = 17$, and one adaptive vote taker

FIG. 4.3. PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM CONTAINING ADAPTIVE VOTE TAKERS COMPARED WITH THE PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM USING m MAJORITY VOTE TAKERS.



D. For $n = 32$, $2n + 1 = 65$, and one adaptive vote taker

FIG. 4.3. PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM CONTAINING ADAPTIVE VOTE TAKERS COMPARED WITH THE PROBABILITY OF FAILURE FOR A REDUNDANT SYSTEM USING m MAJORITY VOTE TAKERS.

place at a smaller value of q . Also, it is seen that for a fixed n the cross-over can be moved back (i.e., toward smaller t and smaller q) by increasing m .

We will now compare the reliability of a system using one adaptive vote taker and having redundancy $2n+1 = 5$ with a system using $m = 10$ majority vote takers and having redundancy $2n+1 = 5$. From the curves of Fig. 4.3A it is seen that

- 1) If the MTF is taken as the criterion for reliability, then the system using $m = 10$ majority vote takers is the most reliable.
- 2) If T_{Δ} is taken as the criterion for reliability, then the system using 10 majority vote takers is the most reliable of the two systems, if $\Delta > 10^{-6}$; and it is the least reliable of the two, if $\Delta < 10^{-6}$.
- 3) If the mission time T is fixed and $Q(T)$ is the criterion for reliability, then the system using 10 majority vote takers is the most reliable of the two systems, if $T > 0.02 Lt$, and it is the least reliable of the two if $T < 0.02 Lt$.

Similar conclusions can be reached for other values of m and n by means of the curves shown in Fig. 4.3. The curves shown in Fig. 4.3 are all based on the use of "ideal" vote takers. If the vote takers are not perfectly reliable, we may use redundant vote takers as discussed in Chapter III. However, in the case of the adaptive vote taker, "ideal" not only refers to the reliability of the circuitry, but also implies instantaneous adaption. With a finite time delay in the adaptive process, the probability of failure $q_n(t)$ will be somewhat larger than shown in the curves of Fig. 4.3, and the cross-over point between q_{na} and Q_n will move further to the left. By increasing m it is possible to make the two curves q_{na} and Q_n cross over at a point where the probability of failure is arbitrarily small.

Finally, we will compare the case of a system using m_a adaptive vote takers with a system using $m = m_1 m_a$ majority vote takers. In Fig. 4.3A we have shown curves for $Q_{na}(t, m_a)$ and $Q_n(t, m_1 m_a)$ where $Q_{na}(t)$ is the probability of failure for a system using $m_a = 100$ adaptive vote takers, and where $Q_n(t)$ is the probability of failure

for a system using $m_l m_a = 1000$ majority vote takers. It is seen that the cross-over for these 2 curves is at $q = 10^{-4}$.

In general, if the cross-over between $q_{na}(t)$ and $Q_n(t)$ is at $q = \Delta$, (where $q_{na}(t)$ is the probability of failure for one adaptive majority group, and $Q_n(t)$ is the probability of failure for the same group containing m majority vote takers), then the cross-over point for the curves $Q_{na}(t, m_a)$ and $Q_n(t, m_a m_l)$ will be at

$$q \approx m_a \cdot \Delta \quad \text{if } m_a \Delta \ll 1$$

From the expressions (4.2) and (4.3) (and Fig. 4.3), it is seen that we can make

$$Q_n(t, m) < q_{na}(t) \quad \text{for all } t \quad (4.4)$$

if we employ about twice the amount of redundancy in the system using majority vote takers as is used in the system using adaptive vote takers. Specifically (4.4) is satisfied if

$$(n+1) \geq 2\eta \quad (4.5)$$

and

$$\binom{2n+1}{n} < m^n (2\eta+1) \quad (4.6)$$

where $(2n+1)$ is the number of redundant circuits in the case of majority logic and $(2\eta+1)$ is the number of redundant circuits in the case of adaptive vote takers. (4.6) is satisfied for $m \geq 3$ if $n \leq 14$.

B. CONCLUSION

Two types of restoring organs have been compared, the adaptive vote taker and the majority vote taker. In a triple redundant system the two types of restoring organs are equivalent. In a system of higher redundancy the adaptive vote taker is considerably more efficient as a restoring organ than the majority vote taker. However, it is always possible to achieve the same system reliability by means of majority

vote takers as can be achieved by means of adaptive vote takers, by using more majority vote takers and/or by using a higher amount of redundancy. Under a wide range of conditions the use of approximately 10 majority vote takers in place of each adaptive vote taker will (for a fixed amount of redundancy) result in a system of superior reliability.

The choice of restoring organ to be used in a given redundant system will then in part depend on the ease with which the particular restoring organ can be realized. At the present time, there is no simple technique for realizing adaptive vote takers, whereas majority vote takers with up to about 9 inputs are relatively easy to implement. Thus, for the time being, the use of majority vote takers appears more promising than the use of adaptive vote takers.

V. CONCLUSIONS

This report presents a quantitative evaluation of the improvement in reliability which can be achieved in a digital system by the use of redundancy and restoring organs. Simple expressions are given for the trade-off between added system complexity (i.e., the number of vote takers and the amount of redundancy) and the corresponding improvement in system reliability.

It is concluded that while the use of redundancy and restoring organs can substantially increase the MTF of a digital system, the technique is much more effective in increasing the useful life T_{Δ} (for $\Delta \ll 1$) of a system. Thus, this technique will be most useful in the case of a system that must operate with an exceedingly small probability of failure for a relatively short period of time.

The improvement in system reliability is found to increase rapidly as a function of the number of restoring organs employed. For a highly redundant system the MTF and T_{Δ} increase almost proportionally with the number of restoring organs. Thus, this technique for improving system reliability will be most useful in the case of large systems, that is in the case of systems which can conveniently be divided into a large number of binary subsystems.

Two types of restoring organs have been considered: the majority vote taker and the adaptive vote taker. For a triple redundant system the two types of restoring organs are equivalent. For systems of higher redundancy the adaptive vote taker is more efficient as a restoring organ than the majority vote taker. However, the reliability which can be achieved by using a given number of adaptive vote takers in a redundant system can often be equalled or exceeded by using instead approximately 10 times as many majority vote takers in the system. The choice of restoring organ will therefore in part depend on the relative cost of implementing the two types of restoring organs.

APPENDIX A. DEVELOPMENT OF A LOWER BOUND ON THE MTF FOR A TRIPLE
REDUNDANT SYSTEM USING m MAJORITY VOTE TAKERS

From (2.3) we have

$$q_n(t) = q_0^3 + 3p_0 q_0^2 = q_0^3 + 3(1 - q_0)q_0^2 \quad \text{for } n = 1 \quad (\text{A.1})$$

$$q_1(t) = 3q_0^2 - 2q_0^3$$

$$p_1(t) = 1 - 3q_0^2 + 2q_0^3 \quad (\text{A.2})$$

$$p_1(t) \geq 1 - 3q_0^2 \quad (\text{A.3})$$

next, if $p_0(t) = e^{-\lambda t}$ then $q_0(t) \leq \lambda t$ and

$$p_1(t) \geq (1 - 3(\lambda t)^2) \quad (\text{A.4})$$

$$p_1(t) = (p_1(t))^m \geq (1 - 3(\lambda t)^2)^m \quad \text{if } (1 - 3(\lambda t)^2) \geq 0 \quad (\text{A.5})$$

Next, we shall show that for $m \geq 3$

$$(1-x)^m = \sum_{i=0}^m (-1)^i \binom{m}{i} x^i \geq 1 - mx + \binom{m}{2} x^2 - \binom{m}{3} x^3$$

$$\text{if } 0 \leq x < \frac{5}{m} \quad (\text{A.6})$$

To show the inequality of (A.6) we show that the remainder

$$\sum_{i=4}^m (-1)^i \binom{m}{i} x^i \geq 0 \quad \text{if } 0 \leq x \leq \frac{5}{m}$$

Observe that the first term of the remainder is positive, and that the ratio of succeeding terms is

$$\frac{\binom{m}{i}}{\binom{m}{i+1}} \frac{x^i}{x^{i+1}} = \frac{i+1}{(m-1)x} > 1 \quad \text{if } 0 < x < \frac{5}{m} \text{ and } i \geq 4 \quad (\text{A.7})$$

thus, the terms are decreasing with i , and since the first term is positive the sum must be positive.

Using (A.6) in (A.5) get

$$P_1(t) \geq 1 - m 3(\lambda t)^2 + \binom{m}{2} (3(\lambda t)^2)^2 - \binom{m}{3} (3(\lambda t)^2)^3$$

$$\text{if } 3(\lambda t)^2 < \frac{5}{m} \quad (\text{A.8})$$

$$\text{MTF} = \int_0^\infty P_1(t) dt \geq \int_0^t P_1(t) dt$$

$$\text{MTF} > \int_0^{t_1} 1 - m 3(\lambda t)^2 + \binom{m}{2} (3(\lambda t)^2)^2 - \binom{m}{3} (3(\lambda t)^2)^3 dt$$

$$\text{if } 3(\lambda t_1)^2 < \frac{5}{m} \text{ and } m \geq 3 \quad (\text{A.9})$$

Since the integrand is negative for $3(\lambda t)^2 = \frac{5}{m}$ and since the cross-over (from positive to negative integrand) is at approximately $3(\lambda t)^2 = \frac{3}{2} \frac{1}{m}$ we evaluate (A.9) for $t_1 = \frac{1}{\lambda} \frac{1}{\sqrt{2m}}$ and get

$$\text{MTF} > \frac{1}{\lambda} \sqrt{\frac{1}{2m}} \left\{ \frac{1}{2} + \frac{m-1}{m} \cdot \frac{9}{40} - \frac{(m-1)(m-2)}{m^2} \cdot \frac{9}{112} \right\} \quad (\text{A.10})$$

$$\text{MTF} > \frac{1}{\lambda} \sqrt{\frac{1}{m}} 0.45 \quad \text{for } m \geq 3 \quad (\text{A.11})$$

For $m = 1$

$$p_1(t) = p_0^3 + 3p_0^2 (1 - p_0) = 3p_0^2 - 2p_0^3 = 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

$$MTF = \int_0^{\infty} p_1(t) dt = \frac{1}{\lambda} \left(\frac{3}{2} - \frac{2}{3} \right) = \frac{1}{\lambda} 0.83 \quad \text{for } m = 1 \quad (A.12)$$

Substituting $L = m\lambda$ in (A.11) and (A.12) we find for all m that

$$MTF > \frac{1}{L} \sqrt{m} 0.45 \quad (A.13)$$

APPENDIX B. DEVELOPMENT OF A LOWER BOUND ON THE MTF FOR A $(2n+1)$
REDUNDANT SYSTEM USING m MAJORITY VOTE TAKERS

The following is a generalization of the development of Appendix A.
From (2.3) we have

$$q_n(t) = \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} q_o^i p_o^{(2n+1-i)} \quad (B.1)$$

$$q_n(t) = \binom{2n+1}{n} q_o^{n+1} \left\{ (1-q_o)^n + \frac{n}{n+2} q_o (1-q_o)^{n-1} + \dots + \frac{1}{\binom{2n+1}{n}} q_o^n \right\} \quad (B.2)$$

$$q_n(t) \leq \binom{2n+1}{n} q_o^{n+1} \left\{ (1-q_o)^n + q_o (1-q_o)^{n-1} + q_o^2 (1-q_o)^{n-2} + \dots + q_o^n \right\} \quad (B.3)$$

Since

$$(q_o + (1-q_o))^n = 1 = \sum_{i=0}^n \binom{n}{i} q_o^i (1-q_o)^{n-i}$$

then

$$1 \geq \sum_{i=0}^n q_o^i (1-q_o)^{n-i} \quad \text{for } 0 \leq q_o \leq 1 \quad (B.4)$$

so that

$$q_n(t) \leq \binom{2n+1}{n} q_o^{n+1} \quad (B.5)$$

and

$$p_n(t) \geq 1 - \binom{2n+1}{n} q_o^{n+1} \quad (B.6)$$

Now if $p_o(t) = e^{-\lambda t}$ then $q_o(t) \leq \lambda t$

$$\text{and } p_n(t) \geq 1 - \binom{2n+1}{n} (\lambda t)^{n+1} \quad (\text{B.7})$$

Also

$$p_n(t) = (p_n(t))^m \geq \left(1 - \binom{2n+1}{n} (\lambda t)^{n+1}\right)^m \quad \text{if } \binom{2n+1}{n} (\lambda t)^{n+1} < 1 \quad (\text{B.8})$$

We introduce the notation

$$\binom{2n+1}{n} = a \quad \text{and} \quad (\lambda t) = y \quad (\text{B.9})$$

then using the result from (A.6) in (B.8) we get

$$p_n(t) \geq 1 - may^{n+1} + \binom{m}{2} (ay^{n+1})^2 - \binom{m}{3} (ay^{n+1})^3 \quad \text{if } 0 \leq ay^{n+1} \leq \frac{5}{m}$$

and $m \geq 3$ (B.10)

Hence

$$\text{MTF} > \int_0^{y_1} \left(1 - may^{n+1} + \binom{m}{2} (ay^{n+1})^2 - \binom{m}{3} (ay^{n+1})^3 \right) \frac{1}{\lambda} dy \quad (\text{B.11})$$

where

$$\frac{1}{\lambda} dy = dt \quad \text{and} \quad y_1 < \left(\frac{5}{am} \right)^{1/n+1}$$

As in Appendix A we evaluate this integral for

$$y_1 = \left(\frac{3}{2am} \right)^{1/n+1}$$

and get

$$MTF > \frac{1}{\lambda} \left(\frac{3}{2am} \right)^{1/n+1} \left(1 - \frac{1}{n+2} \frac{3}{2} + \frac{1}{2n+3} \frac{m-1}{m} \frac{9}{8} - \frac{1}{3n+4} \frac{(m-1)(m-2)}{m^2} \frac{9}{16} \right) \quad (B.12)$$

Inserting $a = \binom{2n+1}{n}$ and $L = m\lambda$ we get

$$MTF > \frac{1}{L} m^{n/n+1} \left[\frac{3}{2 \binom{2n+1}{n}} \right]^{1/n+1} \left(1 - \frac{3}{(n+2)2} + \frac{(m-1)9}{(2n+3)m8} - \frac{(m-1)(m-2)}{(3n+4)m^2} \frac{9}{16} \right) \quad (B.13)$$

so that for $m \gg 1$

$$\underline{\underline{MTF > \frac{1}{L} K_2(n) m^{n/n+1}}} \quad (B.14)$$

where

$$K_2(n) = \left[\frac{3}{2 \binom{2n+1}{n}} \right]^{1/n+1} \left[1 - \frac{3}{2n+4} + \frac{9}{(2n+3)8} - \frac{9}{(3n+4)16} \right] \quad (B.15)$$

$K_2(n)$ is shown in Table 2 for $1 \leq n \leq 10$.

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